# Math 235: Calculus Lab

Prof. Doug Hundley

Whitman College

Week 10

# This Week:

- Optimizing over a family of paths.
- Discussion of the Lab.
- Grading the Lab.

# Schedule for Section A

Next week (April 11th), no classes: Undergrad Conference.

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We'll use half of April 18th to discuss new material, and will leave the second half to work on anything left. Turn in this lab before class, April 25th.

# Schedule for Section B

We'll use half of April 20th to discuss new material, and will leave the second half to work on anything left. Turn in this lab before class, April 27th.

# Last Time

Last time we talked about going around the equator (total $3\pi$ units), and taking three distinct paths (one to the inner circle, then around, then back out) with a path length of $3\pi$, and finally we discussed taking both circles simultaneously.

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Review the $(\beta, \alpha)$ plane...

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Review the $(\beta, \alpha)$ plane...

Recall: To go from point $A$ to point $B$ as $0 \leq t \leq 1$, we use the parametrization:

$$A(1 - t) + Bt$$

# Optimizing over a Family of Paths

Rather than going from $(0,0)$ to $(\pi/2, \pi)$ as we did in Path 3, let $\beta$ go from 0 to an unknown value, $k$ as $\alpha$ runs from 0 to $\pi$.

Path 4 in the $(\beta, \alpha)$ plane:

$$(0,0) \to (k, \pi) \to (?, \pi) \to (\pi, 0)$$

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Path 4A: $(0,0)$ to $(k,\pi)$
Path 4B: $(k,\pi)$ to $(\pi-k,\pi)$
Path 4C: $(\pi-k,\pi)$ to $(\pi,0)$.

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Path 4B: $(k, \pi)$ to $(\pi - k, \pi)$
Path 4C: $(\pi - k, \pi)$ to $(\pi, 0)$.

Make the appropriate changes to the Maple file. What values should we allow $k$ to take?

Once we get the paths:

- Path 4A: `xt:=k*t;` `yt:=Pi*t`
- Path 4B: `xt:= k*(1-t)+(Pi-k)*t`
- Path 4C: `xt:=(Pi-k)*(1-t)+Pi*t` `yt:=Pi*(1-t)`

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- The path length depends on $k$. Plot it!
- Now find the optimal value of the path length!

Sample solution in Maple:

```
xt:=k*t; yt:=Pi*t;
Path3AF:=subs...  (Same as before)
xt:=k*(1-t)+(Pi-k)*t;  yt:=Pi;
Path3BF:=subs...  (Same as before)
xt:=(Pi-k)*(1-t)+Pi*t; yt:=Pi*(1-t);
Path3CF:=subs...  (Same as before)
dP1:=diff([Path3AF],t);
dP2:=diff([Path3BF],t);
dP3:=diff([Path3CF],t);
```

```
Integrand1:=simplify(dP1[1]^2+dP1[2]^2+dP1[3]^2);
Integrand2:=simplify(dP2[1]^2+dP2[2]^2+dP2[3]^2);
Integrand3:=simplify(dP3[1]^2+dP3[2]^2+dP3[3]^2);
PathLength:=Int(sqrt(Integrand1),t=0..1)+
            Int(sqrt(Integrand2),t=0..1)+
            Int(sqrt(Integrand3),t=0..1);
plot(PathLength,k=0..Pi/2);
```

What should your graph be? At $k = 0$? $k = \pi/2$?

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- Find the numerical value of the best path.

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- Find an expression for the derivative.
- Plot the derivative to get an approximate answer.
- Use the approximation in `fsolve`
- Find the numerical value of the best path.

In Maple:

```
dPath:=diff(PathLength,k);
plot(dPath,k=0..Pi/2);
BestK:=fsolve(dPath=0,k=Pi/4..5*Pi/16);
evalf(subs(k=BestK,PathLength));
```

Continuing: Plot the resulting path in Maple

```
Path3A:=subs(k=BestK,[Path3AF]);
Path3B:=subs(k=BestK,[Path3BF]);
Path3C:=subs(k=BestK,[Path3CF]);
P1:=spacecurve(Path3A,t=0..1,color=black,thickness=5):
P2:=spacecurve(Path3B,t=0..1,color=black,thickness=5):
P3:=spacecurve(Path3C,t=0..1,color=black,thickness=5):
display3d(Torus1,P1,P2,P3);
```

# Grading the paper

The paper will be worth 20 points total.

Typesetting the Document (7 pts)

- Use of sections (Introduction, Discussion, Conclusions). Each section is used appropriately to move the paper along.
- General typesetting (includes spelling, general grammar).
- General LaTeX rules followed (use of dollar signs and slashes (for sine and cosine), etc).
- Equations and Figures are numbered and referenced appropriately (use at least 1 numbered equation and 1 figure)
- Use at least one citation (some Calculus book is fine)

Mathematics (6 pts):
The mathematics is clearly explained, correct and complete.

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General flow and completeness of the lab (7 pts).
Includes things like giving a good setup/introduction to the problem, having enough figures, the sections should flow from one to the next, etc.