Math 235: Calculus Lab

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Whitman College

Week 9

New Schedule

Week	Date (Wed/Thu)	Topic
		Spring Break
9	Mar 31-Apr 04	Surfaces and Curves (Short lab 3)
10	Apr 07-Apr 11	Lab 4 (Ants on Donut)
11	Apr 14-Apr 18	Lab 4, cont.
12	Apr 21-Apr 25	Lab 4, cont.
13	Apr 28-May 02	Intro to Beamer
14	May 05-May 09	Talk, continued
	May 16	Give talks.

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I've updated the CLEo gradebook to include Lab 0 and Lab 1. I have put in comments about Lab 2 (I still have to assign points).

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We'll answer the lab questions in Maple, and turn in the Maple worksheet (sans output) a week from Friday (but we won't work on it in class class next).

Next week: We'll be assigning new groups.

Lines in 2 and 3d

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SOLUTION:

$$(1,3,2)(1-t)+(3,4,5)t, \qquad 0 \le t \le 1$$

or

$$x(t)=(1-t)+3t=2t+1 \qquad y(t)=3(1-t)+4t=t+3$$

$$z(t)=2(1-t)+5t=3t+2$$

Curves in Space

We already know how to plot parametrized curves in two dimensions in Maple:

```
plot([x(t), y(t),t=a..b]);
```

so, for example, the following plots the unit circle:

```
plot([cos(t),sin(t),t=0..2*Pi]);
```

In 3d, a curve is defined by:

$$\langle x(t), y(t), z(t) \rangle$$

Maple example:

```
with(plots):
X:=cos(t); Y:=sin(t); Z:=cos(2*t);
spacecurve([X,Y,Z],t=0..3*Pi,color=black,axes=boxed);
```

Arc Length

If we have a curve in two dimensions specified by $y = f(x)$ for $a \leq x \leq b$, then we can determine the arc length using the integral:

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

If the curve is in two dimensions and is specified by: $\langle x(t), y(t) \rangle$ for $a \leq t \leq b$, then the arc length is found by computing:

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

Similarly, if the curve is in three dimensions and is defined by $\langle x(t), y(t), z(t) \rangle$, $a \leq t \leq b$, then the arc length is found by:

$$L = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z(t))^2}\, dt$$

Integration in Maple

```
Digits:=16;
L:=int(exp(x)/x,x=1..7);
```

To force Maple to give a numerical approximation:

```
L:=Int(exp(x)/x,x=1..7);  #Inert form
L1:=evalf(L);   #Numerical approximation of L
```

Surface

A **surface** in three dimensions can be defined directly by

$$z = f(x, y)$$

More generally, a surface is defined as the graph of:

$$\langle x(u, v), y(u, v), z(u, v) \rangle$$

If you have $z = f(x, y)$, then let $x(u, v) = u$, $y(u, v) = v$ and $z(u, v) = f(u, v)$.

Example: Convert the surface $z = x^2 + y^2$ into a parametric surface.

$$\langle u, v, u^2 + v^2 \rangle$$

Plot it in Maple:

```
plot3d([u,v,u^2+v^2],u=-1..1,v=-2..2);
```