Questions from 2.2, 2.3

1. T/F: If $A\mathbf{x} = \mathbf{0}$ has only the trivial solution, then A is row equivalent to I_n.

2. T/F: In order for B to be the inverse of a matrix A, then both $AB = I$ and $BA = I$ must be true.

3. We said that $AB = AC$ does NOT necessarily imply that $B = C$. What if A is invertible?

4. If $AB = I$, is it true that A, B are invertible?

   Come up with an example:

5. (2.3.15) Is it possible for a 4×4 matrix to be invertible when its columns do not span $\mathbb{R}^4$?

6. (2.3.16) If an $n \times n$ matrix A is invertible, then the columns of A^T are linearly independent. Why?

7. (2.3.17) Can a square matrix with two identical columns be invertible?

8. (2.3.19) If the columns of 7×7 matrix D are linearly independent, what can be said about the solutions to $D\mathbf{x} = \mathbf{b}$? Why?

9. (2.3.20) If A is 5×5 and $A\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$, is it possible that $A\mathbf{x} = \mathbf{b}$ has more than one solution for some $\mathbf{b}$?

10. (2.3.27) If A, B are $n \times n$ and AB is invertible, show that A is invertible.

    Using the hint, since AB is invertible, there is a matrix W such that $ABW = I$. We can regroup matrices so that $A(BW) = I$, and let $D = BW$. Then there is a matrix D so that $AD = I$, and since A is $n \times n$, the IMT states that A is invertible.