Overview: Linear Algebra

- The final examination is comprehensive but it focuses on the main concepts and topics studied during the semester. Since many topics build on earlier material, a majority of the questions will involve ideas from Chapters 4, 5, and 6.
- The theme of the first two chapters is to look at systems of equations from many different points of view (system of equations, vector equation, matrix equation, and linear transformation). It might help to review the various questions that involve existence and uniqueness of solutions. The third chapter was all about the determinant (and Cramer's Rule).
- There may be a few definitions on the exam. The most important definitions include: The product $A\mathbf{x}$, the product AB, linear transformation, linear independence and dependence (in a general vector space), spanning set, basis, subspace, dimension, rank, orthogonal basis, eigenvector (eigenspace), eigenvalue, least-squares solution, similar matrices. Most recently, we defined an inner product in general, and the inner product on $C[a, b]$.
- We had the four fundamental subspaces of a matrix. Related is the kernel of a linear transformation.
- We looked at Markov chains, and they served to motivate the concept of the eigenvalue and eigenvector.
- Matrix factorizations: QR factorization, diagonalization (as PDP^{-1} and PCP^{-1}, and the more general form).
- A number of questions will require that you give reasons for your answers. These reasons will often involve a reference to a theorem. Theorems that have descriptive names attached to them are usually good candidates for a question. Examples:
  - The Basis Theorem (p. 259),
  - The Rank Theorem (p. 265),
  - The Diagonalization Theorem (p. 320) and its complex variant (p. 340),
  - The Best Approximation Theorem (p. 398),
  - The Orthogonal Decomposition Theorem (p. 395),
  - The Gram-Schmidt Process (p. 404)
  - Of particular importance is the Invertible Matrix Theorem (pp. 129, 194, 267268, 312).
- You won't need a calculator for the exam (no calculators allowed). Most of the computational questions will come in your take-home portion of the exam.
- Be sure to review your old exams and the review sheets for them.