

# Review Questions: Exam 3

Chapter 6, Sections 5.1-5.3

You will be given the table of Laplace transforms.

1. Use the definition of the Laplace transform to determine  $\mathcal{L}(f)$ :

(a)

(b)

$$f(t) = \begin{cases} 3, & 0 \leq t < 2 \\ 6 - t, & t \geq 2 \end{cases}$$

$$f(t) = \begin{cases} e^{-t}, & 0 \leq t < 5 \\ -1, & t \geq 5 \end{cases}$$

2. Check your answers to Problem 1 by rewriting  $f(t)$  using the step (or Heaviside) function, and use the table to compute the corresponding Laplace transform.

3. Determine the Laplace transform:

(a)  $t^2 e^{-9t}$

(d)  $e^{3t} \sin(4t)$

(b)  $e^{2t} - t^3 - \sin(5t)$

(e)  $e^t \delta(t - 3)$

(c)  $u_5(t)(t - 5)^4$

(f)  $t^2 u_4(t)$

4. Find the inverse Laplace transform:

(a)  $\frac{2s - 1}{s^2 - 4s + 6}$

(d)  $\frac{3s - 2}{2s^2 - 16s + 10}$

(b)  $\frac{7}{(s + 3)^3}$

(e)  $(e^{-2s} - e^{-3s}) \frac{1}{s^2 + s - 6}$

(c)  $\frac{e^{-2s}(4s + 2)}{(s - 1)(s + 2)}$

5. For the following differential equations, solve for  $Y(s)$  (the Laplace transform of the solution,  $y(t)$ ). Do not invert the transform.

(a)  $y'' + 2y' + 2y = t^2 + 4t$ ,  $y(0) = 0$ ,  $y'(0) = -1$

(b)  $y'' + 9y = 10e^{2t}$ ,  $y(0) = -1$ ,  $y'(0) = 5$

(c)  $y'' - 4y' + 4y = t^2 e^t$ ,  $y(0) = 0$ ,  $y'(0) = 0$

6. Solve the given initial value problems using Laplace transforms:

(a)  $2y'' + y' + 2y = \delta(t - 5)$ , zero initial conditions.

(b)  $y'' + 6y' + 9y = 0$ ,  $y(0) = -3$ ,  $y'(0) = 10$

(c)  $y'' - 2y' - 3y = u_1(t)$ ,  $y(0) = 0$ ,  $y'(0) = -1$

(d)  $y'' + 4y = \delta(t - \frac{\pi}{2})$ ,  $y(0) = 0$ ,  $y'(0) = 1$

(e)  $y'' + y = \sum_{k=1}^{\infty} \delta(t - 2k\pi)$ ,  $y(0) = y'(0) = 0$ . Write your answer in piecewise form.

