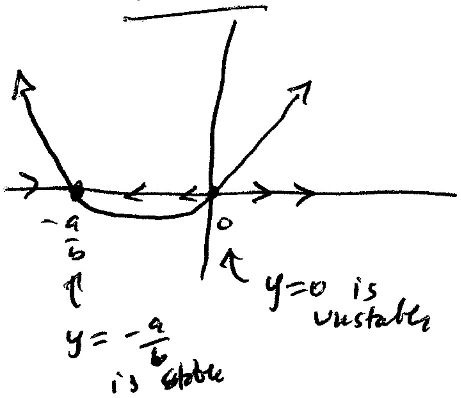
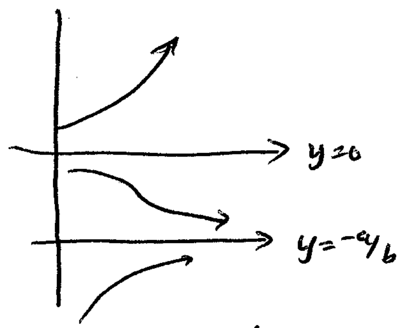
## 2.5

**#1** $y' = ay + by^2 = y(a+by)$

**Phase:**



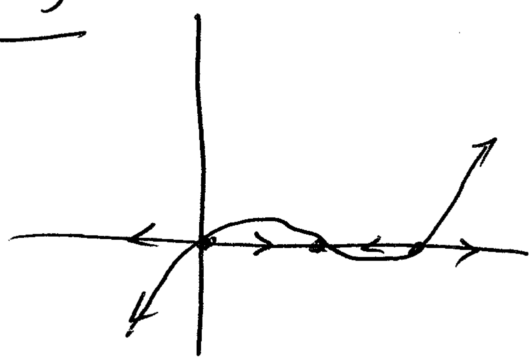

$\Rightarrow$




Direction field.

$\leftarrow$ I included $y_0$ both positive & negative.

**# 3**



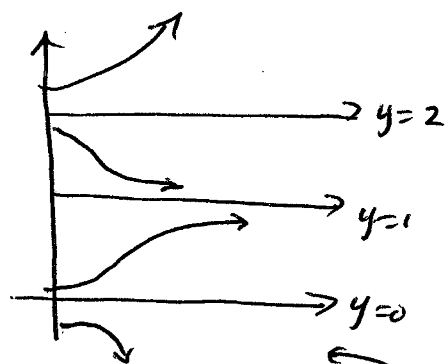
phase

$y=0$ is unstable

$y=1$ is stable

$y=2$ is unstable

$\Rightarrow$

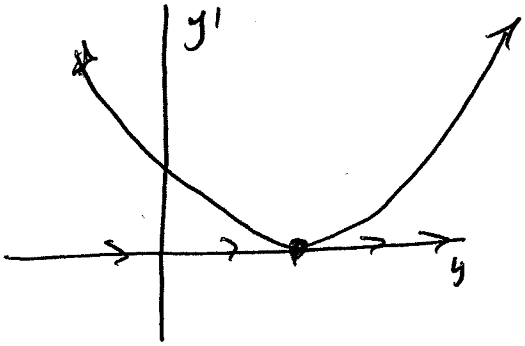



Direction field

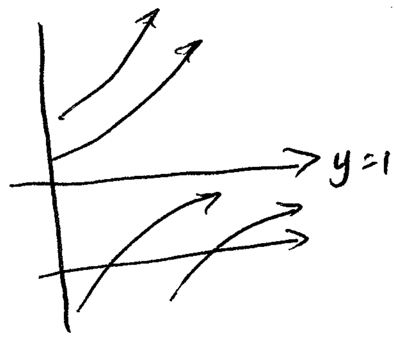
$\leftarrow$ Include $y_0 < 0$

2.5, 7(a) $y' = k(1-y)^2 \Rightarrow (1-y)^2 = 0 \Rightarrow y = 1$

(b)




$\Rightarrow$




(C) Solve the DE with $y(0) = y_0$ (Separate Variables)

$$y(t) = \frac{y_0 kt - kt - y_0}{y_0 kt - kt - 1}$$

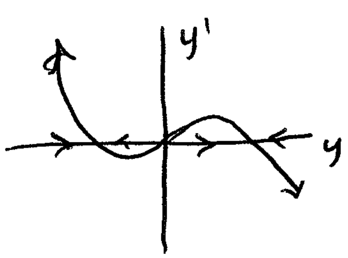
As $t \to \infty$, $y(t) \to 1$. However, note that the denominator ~~has a~~ is zero when:

$$kt(y_0 - 1) - 1 = 0 \Rightarrow t = \frac{1}{k(y_0 - 1)}$$

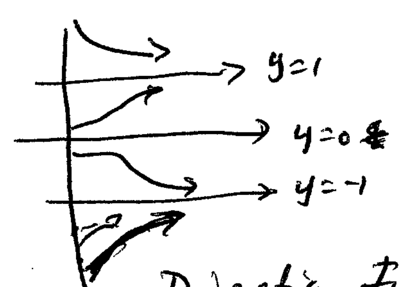
If $y_0 > 1$, this vertical asymptote is in positive time. (Thus, the picture above).

#8: Very similar to 7.

#10: $y' = y(1-y^2) \Rightarrow$ Equilibria at $y = 0$, $y = \pm 1$



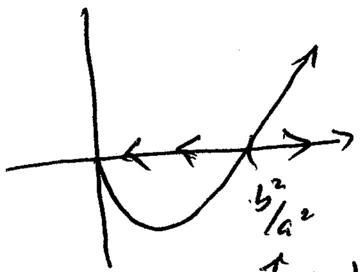

$y = -1$ is stable
$y = 0$ is unstable $\Rightarrow$
$y = 1$ is stable




Direction Field

#11. $y' = ay - b\sqrt{y} = \sqrt{y}\,(a\sqrt{y} - b)$

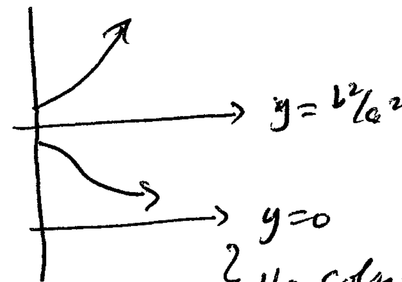
So Equilibria at $y = 0$ and $y = b^2/a^2$




$y=0$ is Stable & $y = b^2/a^2$ is Unstable

Phase Plot

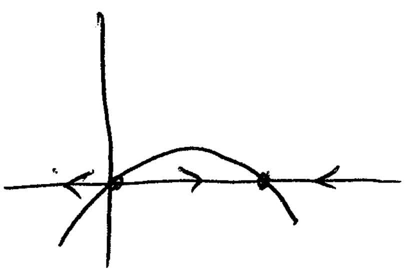
$\Rightarrow$




Direction Field

#22 Analyze as usual: $y' = \alpha y(1-y)$

So equilibria at $y=0$ and $y=1$



$y=0$ is unstable

$y=1$ is stable

Solve the IVP: Done in class... use separation of variables with partial fractions:

$$\int \frac{1}{y(1-y)}\,dy = \int \alpha\,dt \Rightarrow \int \frac{1}{y} + \frac{1}{1-y}\,dy = \alpha t + c$$

$$\ln(y) - \ln|1-y| = \alpha t + c$$

$$\ln\left(\frac{y}{1-y}\right) = \alpha t + c$$

ETC.

#23. We have: $y' = -\beta y$

followed by $x' = -\alpha x y$.

From the first eqn, we see that $y = Ae^{-\beta t}$.

Substitute into the second:

$$x' = -\alpha x A e^{-\beta t} = -\alpha A \cdot x e^{-\beta t} \leftarrow \text{separate variables}$$

$$\int \frac{1}{x}\,dx = -\alpha A \int e^{-\beta t}\,dt$$

$$\ln|x| = \frac{\alpha}{\beta} A e^{-\beta t} + c$$

Solving for $A, C$:

$y = Ae^{-\beta t} \Rightarrow A = y_0$

$$\ln|x| = \frac{\alpha}{\beta} y_0 e^{-\beta t} + c$$

Solve with $x(0) = x_0$

$$\ln|x_0| = \frac{\alpha}{\beta} y_0 + c$$

$$\Rightarrow \quad \ln|x| = \frac{\alpha}{\beta} y_0 e^{-\beta t} + \ln|x_0| - \frac{\alpha}{\beta} y_0$$

So that:

$$x = x_0 e^{\left(\frac{\alpha y_0}{\beta}\left(e^{-\beta t} - 1\right)\right)}$$

So, as $t \to \infty$, $y \to 0$ and $x \to x_0 e^{-\frac{\alpha y_0}{\beta}}$.

(A) Note: This was our first system of DEs!