Chapter 3, Sect 5

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The Set Up

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The general form of the solution is written as:

$$y(t) = y_h(t) + y_p(t)$$

where y_h solves $L(y) = 0$ (the homogeneous part of the solution), and y_p solves $L(y) = g(t)$ (the particular part of the solution).

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Example: $L(y) = y'' - y' - 2y$. Then:

$$L(\mathrm{e}^t \sin(3t)) = 3\mathrm{e}^t \cos(3t) - 11\mathrm{e}^t \sin(3t)$$

$$L(t^2) = 2 - 2t - 2t^2$$

and so on.

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Then $y_p(t) = y_{p_1}(t) + y_{p_2}(t) + y_{p_3}(t)$

Example

Solve: $y'' + 2y' + y = t^2 + e^{2t} - \cos(t)$

- Roots to the char eqn: $r = -1, -1$. Therefore,

$$y_h(t) = e^{-t}(C_1 + C_2 t)$$

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$$-2A\sin(t) + 2B\cos(t) = -\cos(t) \quad \Rightarrow \quad B = -\frac{1}{2}, A = 0$$

Therefore, $y_{p_3}(t) = -\frac{1}{2}\sin(t)$

Example

In conclusion, given $y'' + 2y' + y = t^2 + \mathrm{e}^{2t} - \cos(t)$, the general solution is:

$$y(t) = \mathrm{e}^{-t}(C_1 + C_2 t) + t^2 - 4t + 6 + \frac{1}{9}\mathrm{e}^{2t} - \frac{1}{2}\sin(t)$$

The Method of Undetermined Coefficients

To find the particular solution, we will guess that its form is the same as $g(t)$ (Also see table in text):

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where $s = 0, 1$, or 2.

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$$y(t) = \mathrm{e}^{-t}\left(C_1 + C_2 t + \frac{1}{2}t^2\right)$$

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$$y_{p_2}(t) = At\mathrm{e}^{2t}$$

Substituting, we find: $3A\mathrm{e}^{2t} = \mathrm{e}^{2t}$, so $A = 1/3$. The full solution is

$$y = C_1\mathrm{e}^{-t} + C_2\mathrm{e}^{2t} + (1 + 2t)\mathrm{e}^t + \frac{1}{3}t\mathrm{e}^{2t}$$

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And in fact, the full solution to the DE is:

$$\mathrm{e}^{-t}(C_1 + C_2 t) + \mathrm{e}^t\left(-\frac{1}{8}t + \frac{1}{16}\right)\cos(2t) - \frac{1}{16}\mathrm{e}^t \sin(2t)$$

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3. Could you use complex roots for the previous question?