HW: Extra Practice, Systems of ODEs

1. For each differential equation below, (i) convert into an equivalent system of first order differential equations, (ii) Show that the characteristic equation we get from Chapter 3 is the same as the one for the eigenvalues. (iii) Classify the origin's stability (using Poincare), and (iv) Solve the equation (and the system).

   (a) $y'' - y' - 6y = 0$ (b) $y'' - 2y' + y = 0$ (c) $y'' + 2y' + 5y = 0$

2. For each matrix A below, solve the system $\mathbf{x}' = A\mathbf{x}$ by first converting the system into a second order linear homogeneous DE with constant coefficients (then solve that).

   (a) $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix}$

3. For each matrix A below, give the solution to $\mathbf{x}' = A\mathbf{x}$ using eigenvalues and eigenvectors.

   (a) $\begin{bmatrix} 1 & -1 \\ 1 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} -1 & -1 \\ 0 & -\frac{1}{4} \end{bmatrix}$ (d) $\begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

4. For each system below, (i) Find all equilibrium solutions, (ii) Linearize the DE about each, and (iii) Use the Poincaré Diagram to analyze the stability. Finally, go online to the phase plane plotter and see if your analysis is correct.

   (a) $\begin{aligned} x' &= 1 - y \\ y' &= x^2 - y^2 \end{aligned}$ (b) $\begin{aligned} x' &= \cos(y) \\ y' &= \sin(x) \end{aligned}$ (c) $\begin{aligned} x' &= (2 + x)(y - x) \\ y' &= (4 - x)(y + x) \end{aligned}$

5. Sometimes nonlinear differential equations can be solved by means of methods from Chapter 2, where we convert the system using $dx/dt, dy/dt$ to dy/dx (we've already done a few of these). Here are some more (one linear one just for fun), as a start to your review for the final! Remember, each DE may be classified in multiple ways, so if you get stuck using one technique, you might try another.

   (a) $\begin{aligned} x' &= 2y - 2 \\ y' &= -(2x + 3) \end{aligned}$

   (b) $\begin{aligned} x' &= y(1 - x^3) \\ y' &= x^2 \end{aligned}$

   (c) $\begin{aligned} x' &= x - y \\ y' &= y - 4x \end{aligned}$

   (d) $\begin{aligned} x' &= \mathrm{e}^x \cos(y) + 2\cos(x) \\ y' &= 2y \sin(x) - \mathrm{e}^x \sin(y) \end{aligned}$