Review Questions, Chapters 1-3

1. Define the following terms:

   Differential Equation, Ordinary D.E., Partial D.E., Order of the D.E., Autonomous D.E., Linear First Order D.E., Linear Second Order D.E., Linear Second Order Homogeneous D.E., Direction Field, Implicitly defined function, IVP.

2. Let $y' + p(t)y = g(t)$, $y(t_0) = y_0$. State the Existence and Uniqueness Theorem for this D.E. Also, for what interval is the solution valid?

3. Let $y' = f(t, y)$, $y(t_0) = y_0$. State the Existence and Uniqueness Theorem for this D.E. Also, for what interval is the solution valid?

4. True or False, and explain:

   (a) We cannot draw a direction field for a second order differential equation.

   (b) The solution to $y' = \sin(y)$ cannot be periodic.

   (c) The Existence and Uniqueness Theorem says that $y' = y^{1/3}$, $y(0) = 0$ has a unique solution.

   (d) If $y' = y^2$, then $-y^{-1} = t + C_1$, and $y = -\frac{1}{t} + C_2$

   (e) The functions $3t$ and $|t|$ are linearly independent on $[-1, 1]$, but linearly dependent on $[0, 1]$.

   (f) Let f and g be differentiable. If $W(f, g) = 0$ for all t, then f and g are linearly dependent.

   (g) Let f and g be solutions (valid for all t) to a second order linear differential equation. Then $W(f, g)$ might be zero only at one point.

   (h) $y = 0$ and $y = \cos(t) + \sin(t)$ are both solutions to $y'' + y = 0$. Also, does this violate the Existence and Uniqueness Theorem?

   (i) We can always compute a fundamental set of solutions to $y'' + p(y)y' + q(t)y = 0$.

5. The Wronskian of two functions is $t^2 - 4$. Are the functions linearly independent or linearly dependent? Why?

6. Write a second order linear homogeneous differential equation whose solution is given by:

$$y(t) = \mathrm{e}^{-2t}\left(c_1\cos(3t) + c + 2\sin(3t)\right)$$

7. Give all solutions (in the case of an IVP, give the specific solution).

   (a) $y' = \sqrt{t}\mathrm{e}^{-t} - y$

   (b) $(2y - y^2)\frac{dy}{dt} = t\cos(t)$

   (c) $y'' + 4y' + 5y = 0$, $y(0) = 1$, $y'(0) = 0$.

   (d) $y' = \frac{1}{2}y(3 - y)$

   (e) $y' = 2\cos(3x)$, $y(0) = 2$.

   (f) $y' = 2 + 2t^2 + y + t^2y$

   (g) $y' = xy^2$, $y(0) = y_0$

   (h) $y' = 1 + y^2$

   (i) $2xy^2 + 2y + (2x^2y + 2x)y' = 0$

   (j) $9y'' - 12y' + 4y = 0$, $y(0) = 0$, $y'(0) = -2$.

   (k) $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = -1$

   (l) $y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 1$.

8. Let $y'' + 2y' + y = 0$, $y(0) = 2$, $y'(0) = -3$. Find the solution, and determine if $y(t) = 0$ for any time t. If we change the initial velocity $y'(0)$ to $-\frac{3}{2}$, does $y(t) = 0$ for some time t?

9. Give the general solution in terms of α, and then determine value(s) of α so that $y(t) \to 0$ as $t \to \infty$:

$$y'' - y' - 6y = 0, \qquad y(0) = 1, \quad y'(0) = \alpha$$

10. Determine the longest interval in which the given IVP is certain to have a unique solution. Do not attempt to find the solution:

$$t(t-4)y'' + 3ty' + 4y = 2, \qquad y(3) = 0, \quad y'(3) = -1$$

11. Solve: $y' = ay + b$, where a and b are real values. State the behavior of the solutions as $t \to \infty$ (Note that these will depend on a, b.

12. When we solve $y' = y(3-y)$, $y(0) = -1$, we get the solution:

$$y(t) = \frac{3}{1 - 4e^{-3t}}$$

Plot y' versus y, and determine the behavior of this solution. If we take the limit as $t \to \infty$, $y \to 3$. Does this contradict what we should see in the direction field? Why or why not?

13. A 300 gallon tank contains 100 gallons of brine with a concentration of one pound of salt per gallon. A brine containing one half points of salt per gallon runs into the tank at a rate of four gallons per minute, and the well stirred mixture drains from the tank at the same rate. When is the concentration in the tank 0.7 pounds per gallon?

14. Match the direction fields below with the best statement that describes what you see.

Statements: (1) This is a direction field for an autonomous differential equation. (2) This is a direction field for $y' = f(t)$. (3) The curve shown is not a possible solution. (4) None of these.

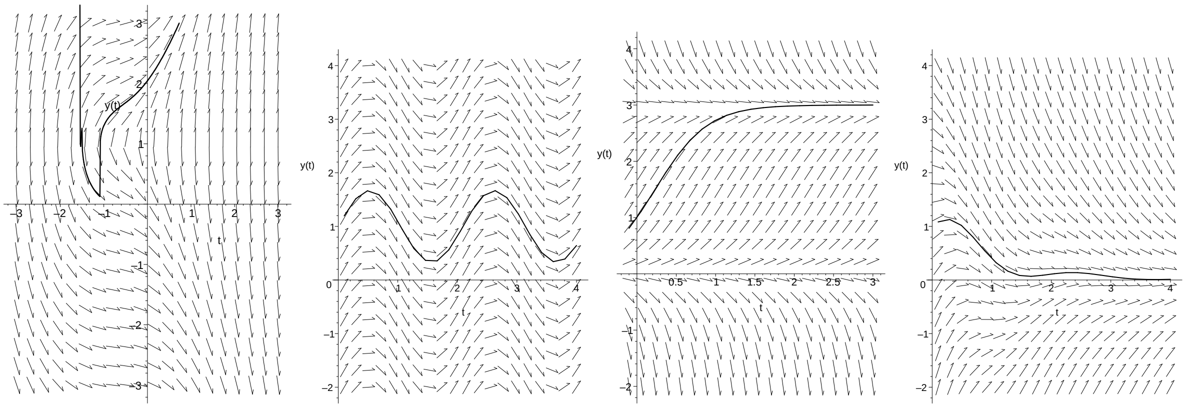

Figure 1: From left to right, direction fields (A)-(D)