Extra Practice: Section 4.4

1. For each differential equation, find the general solution:

   (a) $y'' + 6y' + 9y = \mathrm{e}^{-2t}$

   (b) $y'' + 6y' + 9y = \mathrm{e}^{-3t}$

   (c) $y'' + 6y' + 9y = \cos(3t)$

   (d) $y'' + 6y' + 9y = \sin(3t)$

2. Find the particular part of the solution to the following DE:

$$y'' + py' + qy = \cos(\omega t)$$

3. Suppose that $Y = A\mathrm{e}^{i\omega t} = \dfrac{1}{a+ib}\mathrm{e}^{i\omega t}$.

   Show that the amplitude of the real part of Y is $1/|a+ib|$, and the phase angle (δ) of the real part of Y is the same as the polar angle for $a+ib$.

   HINT: Multiply out the expression for Y first.

4. Find the amplitude and phase angle for the solution to the following DEs. Try to do as little work as you can, and you may use a calculator to assist you, if necessary.

   (a) $y'' + y' + 4y = \cos(2t)$

   (b) $y'' + \frac{1}{2}y' + 2y = \cos(2t)$

5. Given $y'' + \frac{1}{2}y' + 2y = \cos(\omega t)$, find the value of ω that gives the maximum amplitude.