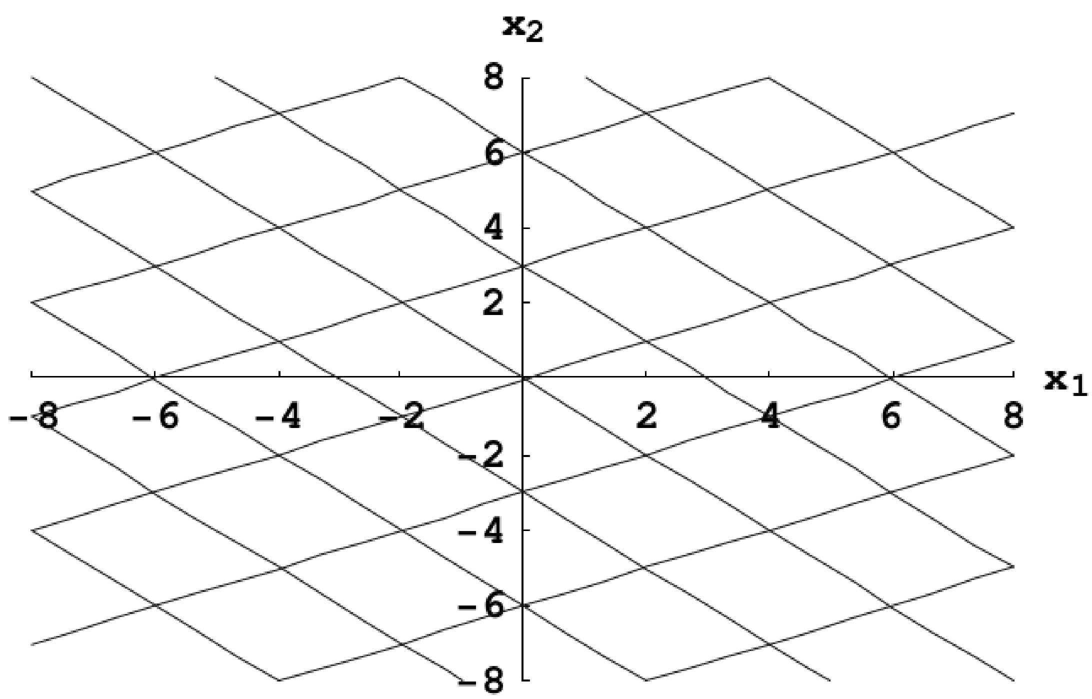
Exercise Set 2 (HW to replace 7.3-7.5)

1. Let $\mathbf{v}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Express each of the following vectors as a linear combination of $\mathbf{v}_1, \mathbf{v}_2$. You're meant to do this graphically, but you might check numerically.

$$\mathbf{a} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}, \qquad \mathbf{c} = \begin{bmatrix} 6 \\ 6 \end{bmatrix}, \qquad \mathbf{d} = \begin{bmatrix} 7 \\ -4 \end{bmatrix}$$



2. Give the general solution to each system $\mathbf{x}' = A\mathbf{x}$ using eigenvalues and eigenvectors. Draw a sketch of the solutions in the (x_1, x_2) plane (or "the phase plane") using the techniques from the video. Finally, classify each as a source, a sink, or a saddle.

   (a) $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$