Exercise Set 1 (HW to replace 7.3-7.5)

This homework is all about solving for eigenvalues and eigenvectors, and we'll also do some visualization and classification of equilibria.

1. Verify that the following function solves the given system of DEs:

$$\mathbf{x}(t) = C_1 e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} \qquad \mathbf{x}' = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix} \mathbf{x}$$

2. For each matrix, find the eigenvalues and eigenvectors. Note that they may be complex (when solving the quadratic equation).

   (a) $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

   (e) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$

   (f) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

3. Convert each of the systems $\mathbf{x}' = A\mathbf{x}$ into a single second order differential equation, and solve it using methods from Chapter 3, if A is given below:

   (a) $A = \begin{bmatrix} 1 & 2 \\ -5 & -1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

4. Give the general solution to each system $\mathbf{x}' = A\mathbf{x}$ using eigenvalues and eigenvectors.

   (a) $A = \begin{bmatrix} 1 & 5 \\ 5 & 1 \end{bmatrix}$

   (c) $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$

   (d) $A = \begin{bmatrix} -1 & 0 \\ 3 & -2 \end{bmatrix}$