Exercise Set for Complex Eigenvalues

1. For each matrix below, solve $\mathbf{x}' = A_i\mathbf{x}$, then sketch the solution being careful of the rotation. Label the origin appropriately (spiral sink, spiral source or center).

$$A_1 = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 2 & -5 \\ 1 & -2 \end{bmatrix}, \qquad A_3 = \begin{bmatrix} 3 & -2 \\ 4 & -1 \end{bmatrix}$$

2. Given the eigenvalues and eigenvectors for some matrix A, write the general solution to $\mathbf{x}' = A\mathbf{x}$. Furthermore, classify the origin as a sink, source, spiral sink, spiral source, saddle, and give a sketch of the solution (go ahead and assume CW rotation if needed).

   (a) $\lambda = -1 + 2i \qquad \mathbf{v} = \begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

   (b) $\lambda = -2, 3 \qquad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   (c) $\lambda = -1, -3 \qquad \mathbf{v}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$

   (d) $\lambda = 1 + 3i \qquad \mathbf{v} = \begin{bmatrix} 1 \\ 1-i \end{bmatrix}$

   (e) $\lambda = 2i \qquad \mathbf{v} = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$