24 Random Questions (to help with Exam 1)

1. State the binomial theorem:

2. The probability that rain is followed by rain is 0.8, a sunny day is followed by rain is 0.6. Find the probability that one has two rainy days then two sunny days.

3. Let X be a discrete random variable. Define f, the probability distribution for X. If we have a candidate function f, how do we know if it is a suitable function (there are two conditions that f must satisfy)?

4. How do we compute the following quantity:

$$\binom{7}{3,\ 3,\ 1}$$

   Come up with a question (that is counting something) for which this was the answer.

5. Let:

$$f(x) = k|x-2|, \quad \text{for} \quad x = -1, 0, 1, 3$$

   Find k so that f is a probability distribution function.

6. A balanced die is tossed twice. Let A be the event that an even number comes up on the first toss. Let B be the event that an even number comes up on the second. Let C be the event that the first two tosses gave the same number. Are A, B, C

   - pairwise independent
   - independent?

7. Find the sample mean, median and variance:

$$\{0, 1, 2, 3, 4, 5\}$$

   (NOTE: I won't ask you to do much computation on the exam, since you won't have your calculator- But you should recall the formulas).

8. State Bayes' Theorem.

9. If A, B are independent prove that A' and B' are independent.

10. Show (with some sample numbers) that $P(B|A) + B(B|A')$ may or may not be equal to 1. Prove that $P(A|B) + P(A'|B) = 1$.

11. In a poker game, 5 cards are dealt from a deck of 52. What is the probability of getting a four-of-a-kind?

12. How many ways can a college team playing 10 games end up with 5 wins, 4 losses and a tie?

13. A carton contains 15 light bulbs of which one is defective. In how many ways can you choose 3 of the bulbs and:

    (a) Get the one that is defective?
    (b) Not get the one that is defective?
    (c) Find the probability of choosing the defective bulb.

14. Given that 8% of the population has diabetes, a health department comes in and gives tests. It correctly diagnoses 95% of all persons with diabetes as having the disease, and incorrectly diagnoses 2% of all persons without diabetes as having the disease.

    (a) Find the probability that the health department will diagnose someone in the population as having the disease.

    (b) Find the probability that someone diagnosed by the health department as having the disease actually has it.

15. Define what it means to say that "X is a random variable".

16. Let X be a random variable. Define what the cumulative distribution function, $F(x)$ is.

17. Experiment: Roll three standard dice. Observe the outcomes.

    (a) How many ways can the three dice all come up with the same number of points?

    (b) How many ways can two of the three come up with the same number, but the third die is different?

    (c) How many ways can all three dice come up with different numbers?

18. Verify that $f(x) = \dfrac{2x}{k(k+1)}$ for $x = 1, 2, 3, \ldots k$ can serve as the probability distribution of a random variable with the given range.

19. We have two men and four women making up a committee of three.

    - What is the probability of choosing no men? one man? two men?
    - Can you come up with a probability distribution function (PDF) for this using binomials?

20. How many ways can a bakery distribute its 7 unsold apple pies to 4 food banks? How does this change if every food bank must get at least one?

21. If I have four skirts, seven blouses and three sweaters, how many ways can I pick 2 skirts, three blouses and one sweater to take along on a trip?

22. Two cards are randomly drawn from a deck of 52. Find the probability that both cards will be greater than 3 and less than 8.

23. List the three axioms (or postulates) that define the probability of an event. You may assume that you have a sample space S, and $A, A_1, A_2, A_3, \ldots$ are events in S

24. On Snoqualmie Pass, the probabilities are 0.23 and 0.24 that a truck stopped at the weigh station will have faulty brakes or badly worn tires (respectively). Also, the probability is 0.38 that a truck stopped will have faulty brakes and/or badly worn tires. What is the probability that a truck stopped will have faulty brakes as well as badly worn tires?