Misc. Review Problems, Exam 3
Math 338
Fall 2008

A few notes:

- Sections: 5.1-5.7, 6.1-6.6 (not 6.4), 7.1-7.5
- This should not be viewed as comprehensive- These are meant to be several random problems to help you get problems out of context. In particular, you might be sure to look at homework from the most recent material.
- You will be given the usual list of distributions.
- You may use a calculator on this exam (and on the Final).

Since you have the chart of distributions, it is important that you understand when to use each (for some, it is more clear than others). I may also ask you to show directly that a given distribution has a certain mean and/or variance- Writing an answer from the chart would not give you any points. As usual, if you are ever unsure about a certain question, you should ask me.

1. Draw a flow chart that relates all the different distributions (if such a connection exists). For example, the geometric distribution is a special case of ??? (using what parameters)?

2. The distribution of some weeds across my yard looks like it might be random. Suppose there are 75 dandelions in my rectangular yard, and I divide my yard into a 10×10 grid.

   The following is a summary of what I got- Is the distribution random?

Number of Weeds	0	1	2	34	
Number of squares	47	36	13	3	1

   By the way, if x is the data in the first row and y is the data in the second, then how would we compute the total number of weeds? Total area?

3. Find the moment generating function for the exponential distribution. Use it to verify the mean and variance.

4. Given that the moment generating function of the normal distribution is:

   $$M(t) = \mathrm{e}^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

   if X is a normally distributed rv with mean 4 and standard deviation 2, what is the moment generating function for $Y = -3X + 4$? What is the distribution of Y (i.e., can you recognize the mgf?)

5. Suppose a large stockpile of used pumps contains 20% that are in need of repair. A maintenance worker is sent to the stockpile with three repair kits. He selects pumps at random and tests them one at a time. If the pump works, he sets it aside for future work. If not, he repairs it. Suppose that it takes 10 minutes to test a pump that is in working order, and 30 minutes to test and repair a pump that does not work. Find the mean and variance of the total time it takes the maintenance worker to use the three repair kits.

6. Suppose X has a normal distribution, $n(x; 2, 3)$. Write $P(X \leq 4)$ so that you can look up the needed probabilities in Table III.

7. Using the definition of Γ, compute $\Gamma(5)$. Hint: You might want to use a table for integration by parts, and the Lemma we had in class about polynomials and exponentials.

8. We said that we could use the Poisson distribution to approximate the Binomial distribution. Under what circumstances, and exactly how do we do it (given the parameters for the Binomial, what are the parameters for the Poisson)?

   We said that we could use the normal distribution to approximate the Binomial distribution. Under what circumstances, and exactly how do we do it (given the parameters for the Binomial, what are the parameters for the normal)?

9. If $Y = aX + b$, and $M(t)$ is the mgf of X, find the mgf of Y:

10. A company employs n people. Its income is a Gamma distribution, $g(x; 80\sqrt{n}, 2)$ Its cost is $8n$. Find the number of people that will give the maximum expected profit (Income−Cost).

11. If X is a normal rv, $n(x; 2, 3)$, then use Table III to give:

    - $P(X \geq 4)$
    - $P(X \geq 1)$
    - $P(-1 < X < 4)$

12. Show that the exponential distribution is memoryless. That is, that

$$P(X \geq t + T | X \geq T) = P(X \geq T)$$

13. Show that the geometric distribution is memoryless. That is,

$$P(X \geq t + T | X \geq T) = P(X \geq T)$$

14. Let X_1 be a uniform distribution on the interval $[1, 5]$, let X_2 be a normal distribution, $n(x; 3, 2)$ and let X_3 be another normal distribution, $n(x; -1, 3)$. Find the mean and variance of $Y = 2X_1 - 3x_2 + X_3$, assuming that X_1, X_2, X_3 are independent.

15. If X and Y are two independent rvs having identical gamma distributions, find the joint pdf of the random variables $U = \frac{X}{X+Y}$ and $V = X + Y$.

16. Find the moment generating function for the geometric distribution, and use it to find the first two moments about the origin.

17. Consider two random variables X and Y whose joint pdf is given by: $f(x, y) = \frac{1}{2}$ if $x > 0, y > 0$ and $x + y < 2$ (zero elsewhere). Find the pdf of $U = Y - X$.

18. Let X and Y be iid exponential with $\theta = 1$. Let $Z = \frac{1}{2}(X + Y)$. Find the pdf of Z by the CDF technique.