Chapter 6 Review Solutions

3. Write the LP in normal form, and the optimal tableau is given in the text (to the right):

	$x_1 \geq 0$	$x_2 \geq 0$	$x_3 \geq 0$	rhs
$y_1 \geq 0$	1	1	1	≤ 6
$y_2 \geq 0$	6	0	1	≤ 8
$y_3 \geq 0$	0	1	1	≤ 2
	≥ 5	≥ 1	≥ 2	

x_1	x_2	x_3	s_1	s_2	s_3	rhs
0	1/6	0	0	5/6	7/6	9
0	1/6	0	1	$-1/6$	$-5/6$	3
1	$-1/6$	0	0	1/6	$-1/6$	1
0	1	1	0	0	1	2

(a) For the dual, just go down the columns:

$$\begin{aligned} \min \ w = \ & 6y_1 + 8y_2 + 2y_3 \\ \text{st} \ \ & y_1 + y_2 \geq 5 \\ & y_1 + y_3 \geq 1 \\ & y_1 + y_2 + y_3 \geq 2 \\ & y_1, y_2, y_2 \geq 0 \end{aligned}$$

(b) For the change in c_1, use the row corresponding to x_1. The new Row 0 is therefore:

$$0 \qquad \frac{1-\Delta}{6} \qquad 0 \qquad 0 \qquad \frac{5+\Delta}{6} \qquad \frac{7-\Delta}{6}$$

In order for each of these to remain non-negative, $-5 < \Delta < 1$.

(c) In this case, x_2 is a non-basic variable. We saw last time that the only change to the optimal Row 0 would be in this coefficient, which becomes $\frac{1}{6} - \Delta$, so $\Delta \leq 1/6$.

4. (a) From the LINDO output, we see that the coefficient 300 can increase by 20 or decrease by any amount. Therefore, 310 will keep the current basis, and the new value of z will be

$$\text{New } z = \text{Old } z + (10)(88) = 33,420$$

(b) Look at the Row 3 "RIGHTHAND SIDE RANGES". The allowable increase is 0.4, which is less than the requested increase of 1. Therefore, we cannot answer the question from the LINDO output. (This question wasn't part of the review).

(c) Look at Row 5, an increase of 1 would be allowed, but looking at the SLACK/SURPLUS, we are not currently using all the steel we have. Therefore, we should not buy any more.

(d) Look at the constraint labelled 6). We are allowed to drop it by 2, and the shadow price is -20, so the profit is: $32,540 + (-2)(-20) = 32,580$.

(e) The new column for jeeps (without Row 0) is $\mathbf{a}_4 = [1.2, 0, 2, 4, 0, 0]^T$. The coefficient in Row 0 is 600. The new Row 0 coefficient for this column would be:

$$600 - \mathbf{c}_B^T B^{-1} \mathbf{a}_4$$

Recall that $\mathbf{c}_B^T B^{-1}$ is the solution to the dual, which we can read directly from the LINDO output.

$$600 - [400, 350, 0, 0, -20, 0]\,[1.2, 0, 2, 4, 0, 0]^T = -120$$

Since this is negative, this variable would be brought into the optimal basis. We SHOULD produce jeeps.

6. (a) The dual is straightforward (use the normal form as in 3). The optimal dual solution can be read off of Table 55 (Look at the coefficients for the slack variables):

$$y_1 = 1/2 \qquad y_2 = 1/2$$

(b) If you change b_2, then the optimal changes by (use second column of B^{-1} stored in the column for s_2)

$$\begin{bmatrix} 6 \\ 1 \end{bmatrix} + \Delta \begin{bmatrix} -1 \\ 1/2 \end{bmatrix}$$

Making both of these positive, we see that $-2 < \Delta < 6$. If b_2 is 12, that means $\Delta = 2$, and the new RHS will be $[6 - 2, 1 + 1]^T = [4, 2]^T$, which means $x_1 = 2$ and $x_2 = 4$.

Just to finish it off, the new $z-$value would be found using the shadow price (dual solution):

$$\text{New } z = 9 + 2(1/2) = 10$$

9. (a) The dual is:

$$\begin{aligned} \min \quad w = \quad & 50y_1 + 15y_2 + 10y_3 \\ \text{st} \quad & y_1 + 2y_2 + y_3 \geq 3 \\ & y_1 - y_2 + y_3 \geq 4 \\ & y_1 + y_2 \geq 1 \\ & y_1 \geq 0, y_2 \leq 0, y_3 \text{ urs} \end{aligned}$$

To read the dual from Table 56, consider columns s_1, e_2 (or a_2), and a_3. Therefore, $y_1 = 1$, $y_2 = 0$, $y_3 = 3$. (And the optimal value of w is 80).

(b) For changes in x_1, note that it is a non-basic variable. Therefore, the only change to the optimal Row 0 will be in this column, and it will be $1 - \Delta$. Therefore, $\Delta < 1$.

(c) For changes in x_2, look at the bottom row of $B^{-1}A$ (we can exclude the artificial variables). We see that the only changes will be to x_1, and its new Row 0 coefficient is $1 + \Delta$. Therefore, as long as $\Delta > -1$, the current basis remains optimal.

10. We're getting more practice reading things off of the optimal tableau. In this case,

(a) The dual is straightforward. The solution (reading off of Row 0 from Table 57) is $y_1 = 0, y_2 = 1$.

(b) Changing b_2 means that the optimal RHS becomes:

$$\begin{bmatrix} 4/3 \\ 1/3 \end{bmatrix} + \Delta \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix}$$

So that $-10 < \Delta < 2$. If $b_2 = 5$, then we have $\Delta = -5$, and the new RHS becomes $[14/3, 5/3]^T$ (so $x_1 = 5/3$ and $x_2 = 0$), and the new optimal value is:

$$z = 10 + (1)(-5) = 5$$

16. More practice reading things off the LINDO output. This time it's jewels!

(a) We are allowed to decrease by 100, so the new coefficent of -190 is allowed. The new value of z would be:

$$\text{New } z = 19,000 - (90)(15) = 17,650$$

(b) Look at constraint labeled 6) to see if it is allowed (and it is). Therefore, $19,000 + (-2)(-200) = 19,400$

(c) Labor is constraint labeled 4), and the dual price is \$200.

(d) Sapphires are constraint 3), and the dual price is 0. That is because we have an extra 10 already.

(e) The new column (not including Row 0) for Type 3 would be $\mathbf{a}_4 = [4, 2, 1, 0, 0]$. Labeling the new Row 0 coefficient c_4, then we want to compute

$$-c_4 + \mathbf{c}_B^T B^{-1} \mathbf{a}_4$$

which again is computed using the dual solution:

$$-550 + 100 \cdot 4 + 0 + 200 \cdot 1 + 0 + 0 = 50$$

This new Row 0 coefficient is positive, so it would not have been brought into the optimal basis. Therefore, we would NOT produce any rings of Type 3.

17. The beginning and ending tableaus are shown below:

x_1	x_2	e_1	e_2	rhs
2	1	0	0	0
-1	-1	1	0	-5
-1	2	0	1	-8

$\rightarrow$

x_1	x_2	e_1	e_2	rhs
0	5	0	2	-16
0	-3	1	-1	3
1	-2	0	-1	8

18. (a) The dual would be:

$$\begin{aligned}
\min w &= 6y_1 + 3y_2 + 3y_3 \\
\text{st} \quad 4y_1 + y_2 + 3y_3 &\geq -4 \\
3y_1 + 2y_2 + y_3 &\geq -1 \\
y_1 \leq 0, y_2 &\geq 0, y_3 \text{ URS}
\end{aligned}$$

The solution to the dual is $y_1 = 0, y_2 = 1/5, y_3 = -7/5$.

(b) Adding the column $\begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ would result in the new Row 0 value:

$$-c_3 + \mathbf{c}_B^T B^{-1} \mathbf{a}_3 = -c_3 + \mathbf{y}^T \mathbf{a}_3 = 1 + [0, 1/5, -7/5] \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 - 6/5 < 0$$

This is negative, so the current basis would no longer be optimal.

20. Some practice with complementary slackness. Writing the dual, we have:

$$\begin{aligned} \min \ w = \ & 5y_1 + 8y_2 \\ \text{st} \ \ & y_1 + 2y_2 \geq 3 \\ & 2y_1 + 3y_2 \geq 4 \\ & y_1 + y_2 \geq 1 \\ & 2y_1 + 3y_2 \geq 5 \\ & y_1, y_2 \geq 0 \end{aligned}$$

To use complementary slackness, we need to know which constraints are binding and which are not. Try some: Constraints 1 and 2 do not intersect in the first quadrant. Constraints 1 and 3 do not intersect either. Constraints 1 and 4 do intersect in the first quadrant, and give $y_1 = 1$, $y_2 = 1$, and $w = 13$ (and excess variables e_1 and e_4 are 0). Therefore in the primal, $x_2 = 0$ and $x_3 = 0$. Furthermore, the two slack variables are zero, so we solve

$$\begin{aligned} x_1 + 2x_4 \ &= 5 \\ 2x_1 + 3x_4 \ &= 8 \end{aligned} \qquad \Rightarrow \qquad x_1 = 1, \ x_4 = 2$$

Using this solution, $z = 13$ as well, so we know that this is an optimal solution.

23. More practice reading things off the optimal tableau.

(a) The dual is straightforward. Its optimal solution (from Row 0 of Table 63) is

$$y_1 = 20, \qquad y_2 = 0 \qquad y_3 = 10$$

(b) Changing the price of Ale, use the bottom row of the tableau to get the new Row 0:

$$0 \qquad 0 \qquad 20 - \Delta/3 \qquad 0 \qquad 10 + 2\Delta/3$$

Therefore, the current basis remains optimal if $-15 < \Delta < 60$. Therefore, the price P can range: $25 < P < 100$.

(c) Same question, but now for Beer (use the first row). The optimal Row 0 becomes:

$$0 \qquad 0 \qquad 20 + 2\Delta/3 \qquad 0 \qquad 10 - \Delta/3$$

Therefore, $-30 < \Delta < 30$, and price P for beer is: $20 < P < 80$.

(d) How much can the amount of corn vary? Corn is the first constraint RHS, or b_1. Changing this quantity gives the optimal RHS as:

$$\frac{1}{3} \begin{bmatrix} 40 \\ 10 \\ 40 \end{bmatrix} + \frac{\Delta}{3} \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solving, we get: $-20 < \Delta < 10$. Therefore, the amount of corn can drop by 20 lbs or increase by 10 lbs and we will keep the current optimal basis.

(e) Hops is the second constraint, so:

$$\frac{1}{3} \begin{bmatrix} 40 \\ 10 \\ 40 \end{bmatrix} + \Delta \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Therefore, $\Delta > -4/3$ (and we can increase by any amount).

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(f) For malt, we do the same as the previous 2 items:

$$\frac{1}{3}\begin{bmatrix} 40 \\ 10 \\ 40 \end{bmatrix} + \frac{\Delta}{3}\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \geq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \Rightarrow \quad -20 < \Delta < 10$$

(g) For malt liquor, we would consider adding the column $[0.5, 3, 3]$ to the matrix A, with coefficient of 50 in Row 0. Then the optimal Row 0 coefficient would be (use the dual):

$$-50 + 20(0.5) + 0(3) + 10(3) = -10$$

Since this is negative, we would add malt liquor into the optimal basis (using simplex), so we should manufacture it!

(h) All constraints are scaled by 16, the objective function is the same.

(i) The dual is divided by 16.

Remark: (h) and (i) are interesting side notes, but aren't that important.

33. From the given information, we know that

$$\mathbf{c}_B^T = [c_1, c_2] \quad \text{and} \quad \mathbf{c}_B^T B^{-1} = [1, 2]$$

where the info on the right was taken from the optimal tableau. Transposing both sides, we have:

$$(B^{-1})^T \mathbf{c}_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \Rightarrow \quad (B^T)^{-1}\mathbf{c}_B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \Rightarrow \quad \mathbf{c}_B = B^T \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \end{bmatrix}$$

Therefore, $c_1 = 7$ and $c_2 = 10$.

34. (a) We'll compute the values in Row 0 by parsing out

$$-\mathbf{c}^T + \mathbf{c}_B^T B^{-1} A$$

We have $\mathbf{c}_B^T = [0, 20, 10]$ (note the columns of the identity in the optimal tableau for the order). To get B^{-1}, we have to know which columns of the original tableau were the three columns of the identity matrix. Some thought will give you the columns (in order) labeled a_1, s_2, a_3. Therefore,

$$B^{-1} = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 1 & -1 & 0 \end{bmatrix} \quad \Rightarrow \quad \mathbf{c}_B^T B^{-1} = [10, 10, 0]$$

Therefore, the coefficient for s_2 is $\mathbf{c}_B^T B^{-1} \vec{e}_2$, or 10.

The coefficient for a_1 is $10 + M$

The coefficient for a_3 is M.

(b) The dual and its solution are:

$$\begin{aligned} \min \quad w = \quad & 150y_1 + 40y_2 + 20y_3 \\ \text{st} \quad & y_1 + y_2 \geq 20 \\ & y_1 + y_3 \geq 10 \\ & y_1 \text{ urs}, y_2 \geq 0, y_3 \leq 0 \end{aligned}$$

$y_1 = 10, y_2 = 10, y_3 = 0$.