REVIEW PROBLEMS

All problems from Sections 5.2 and 5.3 are relevant, along with Chapter 5 Review Problems 1, 2, 6, and 7.

Group A

1 Consider the following LP and its optimal tableau (Table 51):

$$\begin{aligned} \max z &= 4x_1 + x_2 \\ \text{s.t.} \quad x_1 + 2x_2 &= 6 \\ x_1 - x_2 &\ge 3 \\ 2x_1 + x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_3 for which the current basis remains optimal. If $b_3 = 11$, what would be the new optimal solution?

2 For the LP in Problem 1, graphically determine the range of values on c_1 for which the current basis remains optimal. (*Hint:* The feasible region is a line segment.)

3 Consider the following LP and its optimal tableau (Table 52):

$$\begin{aligned} \max z &= 5x_1 + x_2 + 2x_3 \\ \text{s.t.} \quad x_1 + x_2 + x_3 &\le 6 \\ 6x_1 + x_3 &\le 8 \\ x_2 + x_3 &\le 2 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of c_1 for which the current basis remains optimal.

c Find the range of values of c_2 for which the current basis remains optimal.

4 Carco manufactures cars and trucks. Each car contributes \$300 to profit and each truck, \$400. The resources required to manufacture a car and a truck are shown in Table 53. Each day, Carco can rent up to 98 Type 1 machines at a cost of \$50 per machine. The company now has 73 Type 2 machines and 260 tons of steel available. Marketing considerations dictate that at least 88 cars and at least 26 trucks be produced. Let

$$\begin{aligned} \text{X1} &= \text{number of cars produced daily} \\ \text{X2} &= \text{number of trucks produced daily} \\ \text{M1} &= \text{type 1 machines rented daily} \end{aligned}$$

To maximize profit, Carco should solve the LP given in Figure 11. Use the LINDO output to answer the following questions:

a If cars contributed \$310 to profit, what would be the new optimal solution to the problem?

b What is the most that Carco should be willing to pay to rent an additional Type 1 machine for 1 day?

c What is the most that Carco should be willing to pay for an extra ton of steel?

d If Carco were required to produce at least 86 cars, what would Carco's profit become?

e Carco is considering producing jeeps. A jeep contributes \$600 to profit and requires 1.2 days on machine 1, 2 days on machine 2, and 4 tons of steel. Should Carco produce any jeeps?

5 The following LP has the optimal tableau shown in Table 54.

$$\begin{aligned} \max z &= 4x_1 + x_2 \\ \text{s.t.} \quad 3x_1 + x_2 &\ge 6 \\ 2x_1 + x_2 &\ge 4 \\ x_1 + x_2 &= 3 \\ x_1, x_2 &\ge 0 \end{aligned}$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of the objective function coefficient of x_2 for which the current basis remains optimal.

c Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal.

6 Consider the following LP and its optimal tableau (Table 55):

$$\begin{aligned} \max z &= 3x_1 + x_2 - x_3 \\ \text{s.t.} \quad 2x_1 + x_2 + x_3 &\le 8 \\ 4x_1 + x_2 - x_3 &\le 10 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

TABLE 51

z	x_1	x_2	e_2	s_3	a_1	a_2	rhs
1	0	0	0	$\frac{7}{3}$	$M - \frac{2}{3}$	M	$\frac{58}{3}$
0	0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	0	$\frac{2}{3}$
0	1	0	0	$\frac{2}{3}$	$-\frac{1}{3}$	0	$\frac{14}{3}$
0	0	0	1	1	-1	-1	1

TABLE 52

z	x_1	x_2	x_3	s_1	s_2	s_3	rhs
1	0	$\frac{1}{6}$	0	0	$\frac{5}{6}$	$\frac{7}{6}$	9
0	0	$\frac{1}{6}$	0	1	$-\frac{1}{6}$	$-\frac{5}{6}$	3
0	1	$-\frac{1}{6}$	0	0	$\frac{1}{6}$	$-\frac{1}{6}$	1
0	0	1	1	0	0	1	2

TABLE 53

Vehicle	Days on Type 1 Machine	Days on Type 2 Machine	Tons of Steel
Car	0.8	0.6	2
Truck	1	0.7	3

FIGURE 11

LINDO Output for Carco (Problem 4)

```
MAX          300 X1 + 400 X2 - 50 M1
SUBJECT TO
       2)    0.8 X1 + X2 - M1 <=       0
       3)     M1 <=  98
       4)    0.6 X1 + 0.7 X2 <=   73
       5)    2 X1 + 3 X2 <=   260
       6)     X1 >=   88
       7)     X2 >=   26
END

    LP OPTIMUM FOUND  AT STEP    1

         OBJECTIVE FUNCTION VALUE

   1)        32540.0000

VARIABLE          VALUE         REDUCED COST
       X1         88.000000         0.000000
       X2         27.599998         0.000000
       M1         98.000000         0.000000

   ROW        SLACK OR SURPLUS    DUAL PRICES
       2)          0.000000       400.000000
       3)          0.000000       350.000000
       4)          0.879999         0.000000
       5)          1.200003         0.000000
       6)          0.000000       -20.000000
       7)          1.599999         0.000000

NO. ITERATIONS=          1

   RANGES IN WHICH THE BASIS IS UNCHANGED

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE          ALLOWABLE
                  COEF          INCREASE           DECREASE
      X1       300.000000       20.000000          INFINITY
      X2       400.000000        INFINITY          25.000000
      M1       -50.000000        INFINITY         350.000000

                          RIGHTHAND SIDE RANGES
    ROW          CURRENT        ALLOWABLE          ALLOWABLE
                   RHS          INCREASE           DECREASE
       2         0.000000        0.400001           1.599999
       3        98.000000        0.400001           1.599999
       4        73.000000        INFINITY           0.879999
       5       260.000000        INFINITY           1.200003
       6        88.000000        1.999999           3.000008
       7        26.000000        1.599999          INFINITY
```

TABLE 54

z	x_1	x_2	e_1	e_2	a_1	a_2	a_3	rhs
1	0	3	0	0	M	M	$M + 4$	12
0	1	1	0	0	0	0	1	3
0	0	2	1	0	-1	0	3	3
0	0	1	0	1	0	-1	2	2

TABLE 55

z	x_1	x_2	x_3	s_1	s_2	rhs
1	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	9
0	0	1	3	2	-1	6
0	1	0	-1	$-\frac{1}{2}$	$\frac{1}{2}$	1

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_2 for which the current basis remains optimal. If $b_2 = 12$, what is the new optimal solution?

7 Consider the following LP:

$$\max z = 3x_1 + 4x_2$$
$$\text{s.t.} \quad 2x_1 + x_2 \leq 8$$
$$4x_1 + x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

The optimal solution to this LP is $z = 32$, $x_1 = 0$, $x_2 = 8$, $s_1 = 0$, $s_2 = 2$. Graphically find the range of values of c_1 for which the current basis remains optimal.

8 Wivco produces product 1 and product 2 by processing raw material. As much as 90 lb of raw material may be purchased at a cost of \$10/lb. One pound of raw material can be used to produce either 1 lb of product 1 or 0.33 lb of product 2. Using a pound of raw material to produce a pound of product 1 requires 2 hours of labor or 3 hours to produce 0.33 lb of product 2. A total of 200 hours of labor are available, and at most 40 pounds of product 2 can be sold. Product 1 sells for \$13/lb, and product 2 sells for \$40/lb. Let

RM = pounds of raw material processed

P1 = pounds of raw material used to produce product 1

P2 = pounds of raw material used to produce product 2

To maximize profit, Wivco should solve the following LP:

$$
\begin{aligned}
\max z &= 13\text{P1} + 40(0.33)\text{P2} - 10\text{RM} \\
\text{s.t.} \quad \text{RM} &\geq \text{P1} + \text{P2} \\
2\text{P1} + 3\text{P2} &\leq 200 \\
\text{RM} &\leq 90 \\
0.33\text{P2} &\leq 40 \\
\text{P1}, \text{P2}, \text{RM} &\geq 0
\end{aligned}
$$

Use the LINDO output in Figure 12 to answer the following questions:

a If only 87 lb of raw material could be purchased, what would be Wivco's profits?

b If product 2 sold for \$39.50/lb, what would be the new optimal solution?

c What is the most that Wivco should pay for another pound of raw material?

d What is the most that Wivco should pay for another hour of labor?

e Suppose that 1 lb of raw material could also be used to produce 0.8 lb of product 3, which sells for \$24/lb. Processing 1 lb of raw material into 0.8 lb of product 3 requires 7 hours of labor. Should Wivco produce any of product 3?

9 Consider the following LP and its optimal tableau (Table 56):

$$
\begin{aligned}
\max z &= 3x_1 + 4x_2 + x_3 \\
\text{s.t.} \quad x_1 + x_2 + x_3 &\leq 50 \\
2x_1 - x_2 + x_3 &\geq 15 \\
x_1 + x_2 &= 10 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal.

c Find the range of values of the objective function coefficient for x_2 for which the current basis remains optimal.

FIGURE 12

LINDO Output for Wivco (Problem 8)

```
MAX     13 P1 + 13.2 P2 - 10 RM
SUBJECT TO
       2) - P1 - P2 + RM >= 0
       3)  2 P1 + 3 P2 <=   200
       4)   RM <=   90
       5)  0.33 P2 <=  40
END

    LP OPTIMUM FOUND  AT STEP    3

         OBJECTIVE FUNCTION VALUE

 1)         274.000000

VARIABLE          VALUE          REDUCED COST
       P1        70.000000           0.000000
       P2        20.000000           0.000000
       RM        90.000000           0.000000

    ROW    SLACK OR SURPLUS      DUAL PRICES
       2)        0.000000         -12.600000
       3)        0.000000           0.200000
       4)        0.000000           2.600000
       5)       33.400002           0.000000

NO. ITERATIONS=      3

   RANGES IN WHICH THE BASIS IS UNCHANGED

                           OBJ COEFFICIENT RANGES
VARIABLE          CURRENT        ALLOWABLE         ALLOWABLE
                   COEF          INCREASE          DECREASE
      P1        13.000000         0.200000          0.866667
      P2        13.200000         1.300000          0.200000
      RM       -10.000000        INFINITY           2.600000

                           RIGHTHAND SIDE RANGES
     ROW          CURRENT        ALLOWABLE         ALLOWABLE
                    RHS          INCREASE          DECREASE
       2          0.000000        23.333334         10.000000
       3        200.000000        70.000000         20.000000
       4         90.000000        10.000000         23.333334
       5         40.000000        INFINITY          33.400002
```

TABLE 56

z	x_1	x_2	x_3	s_1	e_2	a_2	a_3	rhs
1	1	0	0	1	0	M	$M+3$	80
0	-3	0	0	1	1	-1	-2	15
0	0	0	1	1	0	0	-2	40
0	1	1	0	0	0	0	1	10

TABLE 57

z	x_1	x_2	s_1	s_2	rhs
1	0	0	0	1	10
0	0	$\frac{1}{3}$	1	$-\frac{2}{3}$	$\frac{4}{3}$
0	1	$\frac{7}{3}$	0	$\frac{1}{3}$	$\frac{10}{3}$

10 Consider the following LP and its optimal tableau (Table 57):

$$\begin{aligned} \max z &= 3x_1 + 2x_2 \\ \text{s.t.} \quad 2x_1 + 5x_2 &\le 8 \\ 3x_1 + 7x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

a Find the dual of this LP and its optimal solution.

b Find the range of values of b_2 for which the current basis remains optimal. Also find the new optimal solution if $b_2 = 5$.

11 Consider the following LP:

$$\begin{aligned} \max z &= 3x_1 + x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\le 8 \\ 4x_1 + x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

The optimal solution to this LP is $z = 9$, $x_1 = 1$, $x_2 = 6$. Graphically find the range of values of b_2 for which the current basis remains optimal.

12 Farmer Leary grows wheat and corn on his 45-acre farm. He can sell at most 140 bushels of wheat and 120 bushels of corn. Each planted acre yields either 5 bushels of wheat or 4 bushels of corn. Wheat sells for \$30 per bushel, and corn sells for \$50 per bushel. Six hours of labor are needed to harvest an acre of wheat, and 10 hours are needed to harvest an acre of corn. As many as 350 hours of labor can be purchased at \$10 per hour. Let

A1 = acres planted with wheat

A2 = acres planted with corn

L = hours of labor that are purchased

To maximize profits, farmer Leary should solve the following LP:

$$\begin{aligned} \max z &= 150\text{A1} + 200\text{A2} - 10\text{L} \\ \text{s.t.} \quad \text{A1} + \text{A2} &\le 45 \\ 6\text{A1} + 10\text{A2} - \text{L} &\le 0 \\ \text{L} &\le 350 \\ 5\text{A1} &\le 140 \\ 4\text{A2} &\le 120 \\ \text{A1}, \text{A2}, \text{L} &\ge 0 \end{aligned}$$

Use the LINDO output in Figure 13 to answer the following questions:

a What is the most that Leary should pay for an additional hour of labor?

b What is the most that Leary should pay for an additional acre of land?

c If only 40 acres of land were available, what would be Leary's profit?

d If the price of wheat dropped to \$26, what would be the new optimal solution?

e Farmer Leary is considering growing barley. Demand for barley is unlimited. An acre yields 4 bushels of barley and requires 3 hours of labor. If barley sells for \$30 per bushel, should Leary produce any barley?

13 Consider the following LP and its optimal tableau (Table 58):

$$\begin{aligned} \max z &= 4x_1 + x_2 + 2x_3 \\ \text{s.t.} \quad 8x_1 + 3x_2 + x_3 &\le 2 \\ 6x_1 + x_2 + x_3 &\le 8 \\ x_1, x_2, x_3 &\ge 0 \end{aligned}$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of the objective function coefficient of x_3 for which the current basis remains optimal.

c Find the range of values of the objective function coefficient of x_1 for which the current basis remains optimal.

14 Consider the following LP and its optimal tableau (Table 59):

$$\begin{aligned} \max z &= 3x_1 + x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\le 4 \\ 3x_1 + 2x_2 &\ge 6 \\ 4x_1 + 2x_2 &= 7 \\ x_1 \ge 0, x_2 &\ge 0 \end{aligned}$$

a Find the dual to this LP and its optimal solution.

b Find the range of values of the right-hand side of the third constraint for which the current basis remains optimal. Also find the new optimal solution if the right-hand side of the third constraint were $\frac{15}{2}$.

15 Consider the following LP:

$$\begin{aligned} \max z &= 3x_1 + x_2 \\ \text{s.t.} \quad 4x_1 + x_2 &\le 7 \\ 5x_1 + 2x_2 &\le 10 \\ x_1, x_2 &\ge 0 \end{aligned}$$

The optimal solution to this LP is $z = \frac{17}{3}$, $x_1 = \frac{4}{3}$, $x_2 = \frac{5}{3}$. Use the graphical approach to determine the range of values for the right-hand side of the second constraint for which the current basis remains optimal.

16 Zales Jewelers uses rubies and sapphires to produce two types of rings. A Type 1 ring requires 2 rubies, 3 sapphires, and 1 hour of jeweler's labor. A Type 2 ring requires 3 rubies, 2 sapphires, and 2 hours of jeweler's labor. Each Type 1 ring sells for \$400, and each Type 2 ring sells for \$500. All rings produced by Zales can be sold. Zales now has 100 rubies, 120 sapphires, and 70 hours of jeweler's

FIGURE 13

LINDO Output for Wheat/Corn (Problem 12)

```
MAX 150A1+200A2-10L
ST
A1+A2<45
6A1+10A2-L<0
L<350
5A1<140
4A2<120
END


LP OPTIMUM FOUND AT STEP    4

     OBJECTIVE FUNCTION VALUE

     1)    4250.000

  VARIABLE       VALUE        REDUCED COST
     A1       25.000000        0.000000
     A2       20.000000        0.000000
     L       350.000000        0.000000


     ROW   SLACK OR SURPLUS    DUAL PRICES
      2)       0.000000        75.000000
      3)       0.000000        12.500000
      4)       0.000000         2.500000
      5)      15.000000         0.000000
      6)      40.000000         0.000000

  NO. ITERATIONS=      4


  RANGES IN WHICH THE BASIS IS UNCHANGED:

                        OBJ COEFFICIENT RANGES
  VARIABLE       CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
     A1       150.000000       10.000000        30.000000
     A2       200.000000       50.000000        10.000000
     L        -10.000000       INFINITY          2.500000

                        RIGHTHAND SIDE RANGES
     ROW         CURRENT        ALLOWABLE        ALLOWABLE
                   RHS          INCREASE         DECREASE
      2        45.000000        1.200000         6.666667
      3         0.000000       40.000000        12.000000
      4       350.000000       40.000000        12.000000
      5       140.000000       INFINITY         15.000000
      6       120.000000       INFINITY         40.000000
```

TABLE 58

z	x_1	x_2	x_3	s_1	s_2	rhs
1	8	1	0	0	2	16
0	2	2	0	1	-1	4
0	6	1	1	0	1	8

TABLE 59

z	x_1	x_2	s_1	e_2	a_2	a_3	rhs
1	0	0	0	1	$M-1$	$M+\frac{3}{2}$	$\frac{9}{2}$
0	0	0	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$
0	0	1	0	-2	2	$-\frac{3}{2}$	$\frac{3}{2}$
0	1	0	0	1	-1	1	1

labor. Extra rubies can be purchased at a cost of \$100 per ruby. Market demand requires that the company produce at least 20 Type 1 rings and at least 25 Type 2 rings. To maximize profit, Zales should solve the following LP:

X1 = Type 1 rings produced
X2 = Type 2 rings produced
R = number of rubies purchased

$$\begin{aligned}
\max z &= 400\text{X1} + 500\text{X2} - 100\text{R} \\
\text{s.t.} \quad & 2\text{X1} + 3\text{X2} - \text{R} \le 100 \\
& 3\text{X1} + 2\text{X2} \le 120 \\
& \text{X1} + 2\text{X2} \le 70 \\
& \text{X1} \ge 20 \\
& \text{X2} \ge 25 \\
& \text{X1}, \text{X2} \ge 0
\end{aligned}$$

Use the LINDO output in Figure 14 to answer the following questions:

a Suppose that instead of \$100, each ruby costs \$190. Would Zales still purchase rubies? What would be the new optimal solution to the problem?

FIGURE 14

LINDO Output for Jewelry (Problem 16)

```
MAX     400 X1 + 500 X2 - 100 R
SUBJECT TO
       2)    2 X1 + 3 X2 - R <= 100
       3)    3 X1 + 2 X2 <=   120
       4)     X1 + 2 X2 <=   70
       5)     X1 >=    20
       6)     X2 >=    25
END

    LP OPTIMUM FOUND    AT STEP   2

         OBJECTIVE FUNCTION VALUE

1)          19000.0000

VARIABLE         VALUE          REDUCED COST
       X1        20.000000          0.000000
       X2        25.000000          0.000000
        R        15.000000          0.000000

   ROW      SLACK OR SURPLUS       DUAL PRICES
       2)        0.000000         100.000000
       3)       10.000000           0.000000
       4)        0.000000         200.000000
       5)        0.000000           0.000000
       6)        0.000000        -200.000000

NO. ITERATIONS=      2

  RANGES IN WHICH THE BASIS IS UNCHANGED

                         OBJ COEFFICIENT RANGES
VARIABLE        CURRENT          ALLOWABLE          ALLOWABLE
                 COEF            INCREASE           DECREASE
      X1       400.000000        INFINITY           100.000000
      X2       500.000000        200.000000         INFINITY
       R      -100.000000        100.000000         100.000000

                         RIGHTHAND SIDE RANGES
    ROW         CURRENT          ALLOWABLE          ALLOWABLE
                  RHS            INCREASE           DECREASE
       2       100.000000         15.000000         INFINITY
       3       120.000000        INFINITY            10.000000
       4        70.000000          3.333333           0.000000
       5        20.000000          0.000000         INFINITY
       6        25.000000          0.000000           2.500000
```

b Suppose that Zales were only required to produce at least 23 Type 2 rings. What would Zales' profit now be?

c What is the most that Zales would be willing to pay for another hour of jeweler's labor?

d What is the most that Zales would be willing to pay for another sapphire?

e Zales is considering producing Type 3 rings. Each Type 3 ring can be sold for \$550 and requires 4 rubies, 2 sapphires, and 1 hour of jeweler's labor. Should Zales produce any Type 3 rings?

17 Use the dual simplex method to solve the following LP:

$$\begin{aligned} \max z &= -2x_1 - x_2 \\ \text{s.t.} \quad x_1 + x_2 &\geq 5 \\ x_1 - 2x_2 &\geq 8 \\ x_1, x_2 &\geq 0 \end{aligned}$$

18 Consider the following LP:

$$\begin{aligned} \max z &= -4x_1 - x_2 \\ \text{s.t.} \quad 4x_1 + 3x_2 &\geq 6 \\ x_1 + 2x_2 &\leq 3 \\ 3x_1 + x_2 &= 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

After subtracting an excess variable e_1 from the first constraint, adding a slack variable s_2 to the second constraint, and adding artificial variables a_1 and a_3 to the first and third constraints, the optimal tableau for this LP is as shown in Table 60.

a Find the dual to this LP and its optimal solution.

b If we changed this LP to

$$\begin{aligned} \max z &= -4x_1 - x_2 - x_3 \\ \text{s.t.} \quad 4x_1 + 3x_2 + x_3 &\geq 6 \\ x_1 + 2x_2 + x_3 &\leq 3 \\ 3x_1 + x_2 + x_3 &= 3 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

would the current optimal solution remain optimal?

TABLE 60

z	x_1	x_2	e_1	s_2	a_1	a_3	rhs
1	0	0	0	$\frac{1}{5}$	M	$M - \frac{7}{5}$	$-\frac{18}{5}$
0	0	1	0	$\frac{3}{5}$	0	$-\frac{1}{5}$	$\frac{6}{5}$
0	1	0	0	$-\frac{1}{5}$	0	$\frac{2}{5}$	$\frac{3}{5}$
0	0	0	1	1	-1	1	0

19 Consider the following LP:

$$\begin{aligned} \max z &= -2x_1 + 6x_2 \\ \text{s.t.} \quad x_1 + x_2 &\ge 2 \\ -x_1 + x_2 &\le 1 \\ x_1, x_2 &\ge 0 \end{aligned}$$

This LP is unbounded. Use this fact to show that the following LP has no feasible solution:

$$\begin{aligned} \min 2y_1 &+ y_2 \\ \text{s.t.} \quad y_1 - y_2 &\ge -2 \\ y_1 + y_2 &\ge 6 \\ y_1 \le 0, y_2 &\ge 0 \end{aligned}$$

20 Use the Theorem of Complementary Slackness to find the optimal solution to the following LP and its dual:

$$\begin{aligned} \max z &= 3x_1 + 4x_2 + x_3 + 5x_4 \\ \text{s.t.} \quad x_1 + 2x_2 + x_3 + 2x_4 &\le 5 \\ 2x_1 + 3x_2 + x_3 + 3x_4 &\le 8 \\ x_1, x_2, x_3, x_4 &\ge 0 \end{aligned}$$

21 $z = 8$, $x_1 = 2$, $x_2 = 0$ is the optimal solution to the following LP:

$$\begin{aligned} \max z &= 4x_1 + x_2 \\ \text{s.t.} \quad 3x_1 + x_2 &\le 6 \\ 5x_1 + 3x_2 &\le 15 \\ x_1, x_2 &\ge 0 \end{aligned}$$

Use the graphical approach to answer the following questions:

a Determine the range of values of c_1 for which the current basis remains optimal.

b Determine the range of values of c_2 for which the current basis remains optimal.

c Determine the range of values of b_1 for which the current basis remains optimal.

d Determine the range of values of b_2 for which the current basis remains optimal.

22 Radioco manufactures two types of radios. The only scarce resource that is needed to produce radios is labor. The company now has two laborers. Laborer 1 is willing to work up to 40 hours per week and is paid \$5 per hour. Laborer 2 is willing to work up to 50 hours per week and is paid \$6 per hour. The price as well as the resources required to build each type of radio are given in Table 61.

a Letting x_i be the number of type i radios produced each week, show that Radioco should solve the following LP (its optimal tableau is given in Table 62):

TABLE 61

Radio 1		Radio 2	
Price (\$)	Resource Required	Price (\$)	Resource Required
25	Laborer 1: 1 hour	22	Laborer 1: 2 hours
	Laborer 2: 2 hours		Laborer 2: 2 hours
	Raw material cost: \$5		Raw material cost: \$4

TABLE 62

z	x_1	x_2	s_1	s_2	rhs
1	0	0	$\frac{1}{3}$	$\frac{4}{3}$	80
0	1	0	$-\frac{1}{3}$	$\frac{2}{3}$	20
0	0	1	$\frac{2}{3}$	$-\frac{1}{3}$	10

$$\begin{aligned} \max z &= 3x_1 + 2x_2 \\ \text{s.t.} \quad x_1 + 2x_2 &\le 40 \\ 2x_1 + x_2 &\le 50 \\ x_1, x_2 &\ge 0 \end{aligned}$$

b For what values of the price of a Type 1 radio would the current basis remain optimal?

c For what values of the price of a Type 2 radio would the current basis remain optimal?

d If laborer 1 were willing to work only 30 hours per week, would the current basis remain optimal?

e If laborer 2 were willing to work as many as 60 hours per week, would the current basis remain optimal?

f If laborer 1 were willing to work an additional hour, what is the most that Radioco should pay?

g If laborer 2 were willing to work only 48 hours, what would Radioco's profits be? Verify your answer by determining the number of radios of each type that would be produced.

h A Type 3 radio is under consideration for production. The specifications of a Type 3 radio are as follows: price, \$30; 2 hours from laborer 1; 2 hours from laborer 2; cost of raw materials, \$3. Should Radioco manufacture any Type 3 radios?

23 Beerco manufactures ale and beer from corn, hops, and malt. Currently, 40 lb of corn, 30 lb of hops, and 40 lb of malt are available. A barrel of ale sells for \$40 and requires 1 lb of corn, 1 lb of hops, and 2 lb of malt. A barrel of beer sells for \$50 and requires 2 lb of corn, 1 lb of hops, and 1 lb of malt. Beerco can sell all ale and beer that is produced. To maximize total sales revenue, Beerco should solve the following LP:

$$\begin{aligned} \max z &= 40\text{ALE} + 50\text{BEER} \\ \text{s.t.} \quad \text{ALE} + 2\text{BEER} &\le 40 \quad \text{(Corn constraint)} \\ \text{ALE} + \text{BEER} &\le 30 \quad \text{(Hops constraint)} \\ 2\text{ALE} + \text{BEER} &\le 40 \quad \text{(Malt constraint)} \end{aligned}$$

TABLE 63

z	Ale	Beer	s_1	s_2	s_3	rhs
1	0	0	20	0	10	1,200
0	0	1	$\frac{2}{3}$	0	$-\frac{1}{3}$	$\frac{40}{3}$
0	0	0	$-\frac{1}{3}$	1	$-\frac{1}{3}$	$\frac{10}{3}$
0	1	0	$-\frac{1}{3}$	0	$\frac{2}{3}$	$\frac{40}{3}$

$$\text{ALE, BEER} \geq 0$$

ALE = barrels of ale produced, and BEER = barrels of beer produced. An optimal tableau for this LP is shown in Table 63.

a Write down the dual to Beerco's LP and find its optimal solution.

b Find the range of values of the price of ale for which the current basis remains optimal.

c Find the range of values of the price of beer for which the current basis remains optimal.

d Find the range of values of the amount of available corn for which the current basis remains optimal.

e Find the range of values of the amount of available hops for which the current basis remains optimal.

f Find the range of values of the amount of available malt for which the current basis remains optimal.

g Suppose Beerco is considering manufacturing malt liquor. A barrel of malt liquor requires 0.5 lb of corn, 3 lb of hops, and 3 lb of malt and sells for \$50. Should Beerco manufacture any malt liquor?

h Suppose we express the Beerco constraints in ounces. Write down the new LP and its dual.

i What is the optimal solution to the dual of the new LP? (*Hint:* Think about what happens to $\mathbf{c}_{\text{BV}}B^{-1}$. Use the idea of shadow prices to explain why the dual to the original LP (pounds) and the dual to the new LP (ounces) should have different optimal solutions.)

Group B

24 Consider the following LP:

$$\begin{aligned}
\max z = -3x_1 + x_2 + 2x_3 & \\
\text{s.t.} \quad x_2 + 2x_3 &\leq 3 \\
-x_1 + 3x_3 &\leq -1 \\
-2x_1 - 3x_2 &\leq -2 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

a Find the dual to this LP and show that it has the same feasible region as the original LP.

b Use weak duality to show that the optimal objective function value for the LP (and its dual) must be 0.

25 Consider the following LP:

$$\begin{aligned}
\max z = 2x_1 + x_2 + x_3 & \\
\text{s.t.} \quad x_1 + x_3 &\leq 1 \\
x_2 + x_3 &\leq 2 \\
x_1 + x_2 &\leq 3 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

It is given that

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

a Show that the basic solution with basic variables x_1, x_2, and x_3 is optimal. Find the optimal solution.

b Write down the dual to this LP and find its optimal solution.

c Show that if we multiply the right-hand side of each constraint by a non-negative constant k, then the new optimal solution is obtained simply by multiplying the value of each variable in the original optimal solution by k.

26 Wivco produces two products: 1 and 2. The relevant data are shown in Table 64. Each week, as many as 400 units of raw material can be purchased at a cost of \$1.50 per unit. The company employs four workers, who work 40 hours per week (their salaries are considered a fixed cost). Workers can be asked to work overtime and are paid \$6 per hour for overtime work. Each week, 320 hours of machine time are available.

In the absence of advertising, 50 units of product 1 and 60 units of product 2 will be demanded each week. Advertising can be used to stimulate demand for each product. Each dollar spent on advertising product 1 increases its demand by 10 units; each dollar spent for product 2 increases its demand by 15 units. At most \$100 can be spent on advertising. Define

P1 = number of units of product 1 produced each week

P2 = number of units of product 2 produced each week

OT = number of hours of overtime labor used each week

RM = number of units of raw material purchased each week

A1 = dollars spent each week on advertising product 1

A2 = dollars spent each week on advertising product 2

Then Wivco should solve the following LP:

$$\begin{aligned}
\max z = 15\text{P1} + 8\text{P2} - 6(\text{OT}) - 1.5\text{RM} & \\
- \text{A1} - \text{A2} & \\
\text{s.t.} \quad \text{P1} - 10\text{A1} &\leq 50 && (1) \\
\text{P2} - 15\text{A2} &\leq 60 && (2) \\
0.75\text{P1} + 0.5\text{P2} &\leq 160 + (\text{OT}) && (3) \\
2\text{P1} + \text{P2} &\leq \text{RM} && (4) \\
\text{RM} &\leq 400 && (5) \\
\text{A1} + \text{A2} &\leq 100 && (6) \\
1.5\text{P1} + 0.8\text{P2} &\leq 320 && (7) \\
\text{All variables non-negative} &
\end{aligned}$$

Use LINDO to solve this LP. Then use the computer output

TABLE 64

	Product 1	Product 2
Selling price	\$15	\$8
Labor required	0.75 hour	0.50 hour
Machine time required	1.5 hours	0.80 hour
Raw material required	2 units	1 unit

to answer the following questions:

a If overtime were only $4 per hour, would Wivco use it?

b If each unit of product 1 sold for $15.50, would the current basis remain optimal? What would be the new optimal solution?

c What is the most that Wivco should be willing to pay for another unit of raw material?

d How much would Wivco be willing to pay for another hour of machine time?

e If each worker were required (as part of the regular workweek) to work 45 hours per week, what would the company's profits be?

f Explain why the shadow price of row (1) is 0.10. (*Hint:* If the right-hand side of (1) were increased from 50 to 51, then in the absence of advertising for product 1, 51 units could now be sold each week.)

g Wivco is considering producing a new product (product 3). Each unit sells for $17 and requires 2 hours of labor, 1 unit of raw material, and 2 hours of machine time. Should Wivco produce any of product 3?

h If each unit of product 2 sold for $10, would the current basis remain optimal?

27 The following question concerns the Rylon example discussed in Section 3.9. After defining

RB = ounces of Regular Brute produced annually
LB = ounces of Luxury Brute produced annually
RC = ounces of Regular Chanelle produced annually
LC = ounces of Luxury Chanelle produced annually
RM = pounds of raw material purchased annually

the LINDO output in Figure 15 was obtained for this problem. Use this output to answer the following questions:

a Interpret the shadow price of each constraint.

b If the price of RB were to increase by 50¢, what would

FIGURE 15

LINDO Output for Brute/Chanelle (Problem 27)

```
MAX      7 RB + 14 LB + 6 RC + 10 LC - 3 RM
SUBJECT TO
       2)     RM <=    4000
       3)    3 LB + 2 LC +   RM <=    6000
       4)     RM + LB - 3 RM =      0
       5)     RC + LC - 4 RM =      0
END

   LP OPTIMUM FOUND   AT STEP     6

        OBJECTIVE FUNCTIONS VALUE

1)         172666.672

VARIABLE         VALUE           REDUCED COST
       RB     11333.333008         0.000000
       LB       666.666687         0.000000
       RC     16000.000000         0.000000
       LC         0.000000         0.666667
       RM      4000.000000         0.000000

    ROW   SLACK OR SURPLUS        DUAL PRICES
       2)        0.000000          39.666668
       3)        0.000000           2.333333
       4)        0.000000           7.000000
       5)        0.000000           6.000000

NO.  ITERATIONS=    6

   RANGES IN WHICH THE BASIS IS UNCHANGED

                          OBJ COEFFICIENT RANGES
VARIABLE          CURRENT        ALLOWABLE         ALLOWABLE
                   COEF          INCREASE          DECREASE
      RB          7.000000        1.000000         11.900001
      LB         14.000000      119.000000          1.000000
      RC          6.000000       INFINITY           0.666667
      LC         10.000000        0.666667        INFINITY
      RM         -3.000000       INFINITY          39.666668

                          RIGHTHAND SIDE RANGES
   ROW            CURRENT        ALLOWABLE         ALLOWABLE
                    RHS          INCREASE          DECREASE
       2        4000.000000     2000.000000       3400.000000
       3        6000.000000    33999.996094       2000.000000
       4           0.000000      INFINITY        11333.333008
       5           0.000000      INFINITY        16000.000000
```

be the new optimal solution to the Rylon problem?

c If 8,000 laboratory hours were available each year, but only 2,000 lb of raw material were available each year, would Rylon's profits increase or decrease? [*Hint:* Use the 100% Rule to show that the current basis remains optimal. Then use reasoning analogous to (34)–(37) to determine the new objective function value.]

d Rylon is considering expanding its laboratory capacity. Two options are under consideration:

Option 1 For a cost of \$10,000 (incurred now), annual laboratory capacity can be increased by 1,000 hours.
Option 2 For a cost of \$200,000 (incurred now), annual laboratory capacity can be increased by 10,000 hours.

Suppose that all other aspects of the problem remain unchanged and that future profits are discounted, with the interest rate being $11\frac{1}{9}$% per year. Which option, if any, should Rylon choose?

e Rylon is considering purchasing a new type of raw material. Unlimited quantities can be purchased at \$8/lb. It requires 3 laboratory hours to process a pound of the new raw material. Each processed pound yields 2 oz of RB and 1 oz of RC. Should Rylon purchase any of the new material?

28 Consider the following two LPs:

$$\begin{aligned} \max z &= c_1x_1 + c_2x_2 \\ \text{s.t.} \quad & a_{11}x_1 + a_{12}x_2 \leq b_1 \\ & a_{21}x_1 + a_{22}x_2 \leq b_2 \\ & x_1, x_2 \geq 0 \end{aligned} \qquad \text{(LP 1)}$$

$$\begin{aligned} \max z &= 100c_1x_1 + 100c_2x_2 \\ \text{s.t.} \quad & 100a_{11}x_1 + 100a_{12}x_2 \leq b_1 \\ & 100a_{21}x_1 + 100a_{22}x_2 \leq b_2 \\ & x_1, x_2 \geq 0 \end{aligned} \qquad \text{(LP 2)}$$

Suppose that BV $= \{x_1, x_2\}$ is an optimal basis for both LPs, and the optimal solution to LP 1 is $x_1 = 50$, $x_2 = 500$, $z = 550$. Also suppose that for LP 1, the shadow price of both Constraint 1 and Constraint 2 $= \frac{100}{3}$. Find the optimal solution to LP 2 and the optimal solution to the dual of LP 2. (*Hint:* If we multiply each number in a matrix by 100, what happens to B^{-1}?)

29 The following questions pertain to the Star Oil capital budgeting example of Section 3.6. The LINDO output for this problem is shown in Figure 16.

a Find and interpret the shadow price for each constraint.

b If the NPV of investment 1 were \$5 million, would the optimal solution to the problem change?

c If the NPV of investment 2 and investment 4 were each decreased by 25%, would the optimal solution to the problem change? (This part requires knowledge of the 100% Rule.)

d Suppose that Star Oil's investment budget were changed to \$50 million at time 0 and \$15 million at time 1. Would Star be better off? (This part requires knowledge of the 100% Rule.)

e Suppose a new investment (investment 6) is available. Investment 6 yields an NPV of \$10 million and requires a cash outflow of \$5 million at time 0 and \$10 million at time 1. Should Star Oil invest any money in investment 6?

30 The following questions pertain to the Finco investment example of Section 3.11. The LINDO output for this problem is shown in Figure 17.

a If Finco has \$2,000 more on hand at time 0, by how much would their time 3 cash increase?

b Observe that if Finco were given a dollar at time 1, the cash available for investment at time 1 would now be $0.5A + 1.2C + 1.08S_0 + 1$. Use this fact and the shadow price of Constraint 2 to determine by how much Finco's time 3 cash position would increase if an extra dollar were available at time 1.

c By how much would Finco's time 3 cash on hand change if Finco were given an extra dollar at time 2?

d If investment D yielded \$1.80 at time 3, would the current basis remain optimal?

e Suppose that a super money market fund yielded 25% for the period between time 0 and time 1. Should Finco invest in this fund at time 0?

f Show that if the investment limitations of \$75,000 on investments A, B, C, and D were all eliminated, the current basis would remain optimal. (Knowledge of the 100% Rule is required for this part.) What would be the new optimal z-value?

g A new investment (investment F) is under consideration. One dollar invested in investment F generates the following cash flows: time 0, $-$\$1.00; time 1, + \$1.10; time 2, + \$0.20; time 3, + \$0.10. Should Finco invest in investment F?

31 In this problem, we discuss how shadow prices can be interpreted for blending problems (see Section 3.8). To illustrate the ideas, we discuss Problem 2 of Section 3.8. If we define

$$\begin{aligned} x_{6J} &= \text{pounds of grade 6 oranges in juice} \\ x_{9J} &= \text{pounds of grade 9 oranges in juice} \\ x_{6B} &= \text{pounds of grade 6 oranges in bags} \\ x_{9B} &= \text{pounds of grade 9 oranges in bags} \end{aligned}$$

then the appropriate formulation is

$$\max z = 0.45(x_{6J} + x_{9J}) + 0.30(x_{6B} + x_{9B})$$

$$\begin{aligned} \text{s.t.} \quad & x_{6J} + x_{6B} \leq 120{,}000 && \text{(Grade 6 constraint)} \\ & x_{9J} + x_{9B} \leq 100{,}000 && \text{(Grade 9 constraint)} \\ (1) \quad & \underline{6x_{6J} + 9x_{9J}} \geq 8 && \text{(Orange Juice constraint)} \\ (2) \quad & \underline{6x_{6B} + 9x_{9B}} \geq 7 && \text{(Bags constraint)} \\ & x_{6J}, x_{9J}, x_{6B}, x_{9B} \geq 0 \end{aligned}$$

Constraints (1) and (2) are examples of blending constraints, because they specify the proportion of grade 6 and grade 9 oranges that must be blended to manufacture orange juice and bags of oranges. It would be useful to determine how a slight change in the standards for orange juice and bags of oranges would affect profit. At the end of this problem, we

FIGURE 16

LINDO Output for Star Oil (Problem 29)

```
MAX     13 X1 + 16 X2 + 16 X3 + 14 X4 + 39 X5
SUBJECT TO
       2)    11 X1 + 53 X2 + 5 X3 + 5 X4 + 29 X5 <= 40
       3)    3 X1 + 6 X2 + 5 X3 + X4 + 34 X5 <= 20
       4)     X1 <=   1
       5)     X2 <=   1
       6)     X3 <=   1
       7)     X4 <=   1
       8)     X5 <=   1
END

     LP OPTIMUM FOUND   AT STEP  5

         OBJECTIVE FUNCTION VALUE

1)          57.4490166

VARIABLE         VALUE          REDUCED COST
       X1         1.000000          0.000000
       X2         0.200860          0.000000
       X3         1.000000          0.000000
       X4         1.000000          0.000000
       X5         0.288084          0.000000

   ROW     SLACK OR SURPLUS      DUAL PRICES
       2)        0.000000          0.190418
       3)        0.000000          0.984644
       4)        0.000000          7.951474
       5)        0.799140          0.000000
       6)        0.000000         10.124693
       7)        0.000000         12.063268
       8)        0.711916          0.000000

NO. ITERATIONS=     5

   RANGES IN WHICH THE BASIS IS UNCHANGED

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE         ALLOWABLE
                  COEF          INCREASE          DECREASE
      X1         13.000000       INFINITY          7.951474
      X2         16.000000        45.104530        9.117648
      X3         16.000000       INFINITY         10.124693
      X4         14.000000       INFINITY         12.063268
      X5         39.000000        51.666668       30.245283

                          RIGHTHAND SIDE RANGES
    ROW          CURRENT        ALLOWABLE         ALLOWABLE
                   RHS          INCREASE          DECREASE
        2        40.000000        38.264709        9.617647
        3        20.000000        11.275863        8.849057
        4         1.000000         1.139373        1.000000
        5         1.000000       INFINITY          0.799140
        6         1.000000         1.995745        1.000000
        7         1.000000         2.319149        1.000000
        8         1.000000       INFINITY          0.711916
```

explain how to use the shadow prices of Constraints (1) and (2) to answer the following questions:

a Suppose that the average grade for orange juice is increased to 8.1. Assuming the current basis remains optimal, by how much would profits change?

b Suppose the average grade requirement for bags of oranges is decreased to 6.9. Assuming the current basis remains optimal, by how much would profits change?

The shadow price for both (1) and (2) is -0.15. The optimal solution is $x_{6J} = 26{,}666.67$, $x_{9J} = 53{,}333.33$, $x_{6B} = 93{,}333.33$, $x_{9B} = 46{,}666.67$. To interpret the shadow prices of blending Constraints (1) and (2), *we assume that a slight change in the quality standard for a product will not significantly change the quantity of the product that is produced.*

Now note that (1) may be written as

$$6x_{6J} + 9x_{9J} \geq 8(x_{6J} + x_{9J}), \qquad \text{or} \qquad -2x_{6J} + x_{9J} \geq 0$$

If the quality standard for orange juice is changed to $8 + \Delta$, then (1) can be written as

$$6x_{6J} + 9x_{9J} \geq (8 + \Delta)\,(x_{6J} + x_{9J})$$

or

$$-2x_{6J} + x_{9J} \geq \Delta(x_{6J} + x_{9J})$$

Because we are assuming that changing orange juice qual-

FIGURE 17

LINDO Output for Finco (Problem 30)

```
MAX     B + 1.9 D + 1.5 E + 1.08  S2
SUBJECT TO
       2)    D + A + C  +  SO =       100000
       3) - B + 0.5 A + 1.2 C + 1.08 SO -   S1 =   0
       4)  0.5 B - E- S2 +  A + 1.08 S1 =   0
       5)    A <=  75000
       6)    B <=  75000
       7)    C <=  75000
       8)    D <=  75000
       9)    E <=  75000
END

   LP OPTIMUM FOUND  AT STEP    8

        OBJECTIVE FUNCTION VALUE

1)         218500.000

VARIABLE        VALUE          REDUCED COST
        B    30000.000000         0.000000
        D    40000.000000         0.000000
        E    75000.000000         0.000000
       S2        0.000000         0.040000
        A    60000.000000         0.000000
        C        0.000000         0.028000
       S0        0.000000         0.215200
       S1        0.000000         0.350400

    ROW    SLACK OR SURPLUS     DUAL PRICES
        2)        0.000000         1.900000
        3)        0.000000        -1.560000
        4)        0.000000        -1.120000
        5)    15000.000000         0.000000
        6)    45000.000000         0.000000
        7)    75000.000000         0.000000
        8)    35000.000000         0.000000
        9)        0.000000         0.380000

NO. ITERATIONS=      8

   RANGES IN WHICH THE BASIS IS UNCHANGED

                          OBJ COEFFICIENT RANGES
VARIABLE         CURRENT        ALLOWABLE        ALLOWABLE
                  COEF          INCREASE         DECREASE
       B          1.000000         0.029167         0.284416
       D          1.900000         0.475000         0.050000
       E          1.500000        INFINITY          0.380000
      S2          1.080000         0.040000        INFINITY
       A          0.000000         0.050000         0.058333
       C          0.000000         0.028000        INFINITY
      S0          0.000000         0.215200        INFINITY
      S1          0.000000         0.350400        INFINITY

                          RIGHTHAND SIDE RANGES
   ROW           CURRENT        ALLOWABLE         ALLOWABLE
                   RHS          INCREASE          DECREASE
      2      100000.000000     35000.000000      40000.000000
      3           0.000000     37500.000000      56250.000000
      4           0.000000     18750.000000      43750.000000
      5       75000.000000       INFINITY        15000.000000
      6       75000.000000       INFINITY        45000.000000
      7       75000.000000       INFINITY        75000.000000
      8       75000.000000       INFINITY        35000.000000
      9       75000.000000     18750.000000      43750.000000
```

ity from 8 to $8 + \Delta$ does not change the amount produced, $x_{6J} + x_{9J}$ will remain equal to 80,000, and (1) will become

$$-2x_{6J} + x_{9J} \geq 80{,}000\Delta$$

Using the definition of shadow price, now answer parts (a) and (b).

32 Ballco manufactures large softballs, regular softballs, and hardballs. Each type of ball requires time in three departments: cutting, sewing, and packaging, as shown in Table 65 (in minutes). Because of marketing considerations, at least 1,000 regular softballs must be produced. Each

TABLE 65

Balls	Cutting Time	Sewing Time	Packaging Time
Regular softballs	15	15	3
Large softballs	10	15	4
Hardballs	8	4	2

regular softball can be sold for \$3, each large softball, for \$5; and each hardball, for \$4. A total of 18,000 minutes of cutting time, 18,000 minutes of sewing time, and 9,000 minutes of packaging time are available. Ballco wants to maximize sales revenue. If we define

RS = number of regular softballs produced

LS = number of large softballs produced

HB = number of hardballs produced

then the appropriate LP is

$$
\begin{array}{llr}
\max z = 3\text{RS} + 5\text{LS} + 4\text{HB} & & \\
\text{s.t.} \quad 15\text{RS} + 10\text{LS} + 8\text{HB} \le 18{,}000 & & \text{(Cutting constraint)} \\
\quad\quad 15\text{RS} + 15\text{LS} + 4\text{HB} \le 18{,}000 & & \text{(Sewing constraint)} \\
\quad\quad 3\text{RS} + 4\text{LS} + 2\text{HB} \le 9{,}000 & & \text{(Packaging constraint)} \\
\quad\quad \text{RS} \ge 1{,}000 & & \text{(Demand constraint)} \\
\quad\quad \text{RS, LS, HB} \ge 0 & &
\end{array}
$$

The optimal tableau for this LP is shown in Table 66.

a Find the dual of the Ballco problem and its optimal solution.

b Show that the Ballco problem has an alternative optimal solution. Find it. How many minutes of sewing time are used by the alternative optimal solution?

c By how much would an increase of 1 minute in the amount of available sewing time increase Ballco's revenue? How can this answer be reconciled with the fact that the sewing constraint is binding? (*Hint:* Look at the answer to part (b).)

d Assuming the current basis remains optimal, how would an increase of 100 in the regular softball requirement affect Ballco's revenue?

33 Consider the following LP:

$$
\begin{aligned}
\max z &= c_1x_1 + c_2x_2 \\
\text{s.t.} \quad & 3x_1 + 4x_2 \le 6 \\
& 2x_1 + 3x_2 \le 4 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

The optimal tableau for this LP is

$$
\begin{aligned}
z \quad + \quad s_1 + 2s_2 &= 14 \\
x_1 \quad + 3s_1 - 4s_2 &= 2 \\
x_2 - 2s_1 + 3s_2 &= 0
\end{aligned}
$$

Without doing any pivots, determine c_1 and c_2.

34 Consider the following LP and its partial optimal tableau (Table 67):

$$
\begin{aligned}
\max z &= 20x_1 + 10x_2 \\
\text{s.t.} \quad & x_1 + x_2 = 150 \\
& x_1 \le 40 \\
& x_2 \ge 20 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

a Complete the optimal tableau.

b Find the dual to this LP and its optimal solution.

35 Consider the following LP and its optimal tableau (Table 68):

$$
\begin{aligned}
\max z &= c_1x_1 + c_2x_2 \\
\text{s.t.} \quad & a_{11}x_1 + a_{12}x_2 \le b_1 \\
& a_{21}x_1 + a_{22}x_2 \le b_2 \\
& x_1, x_2 \ge 0
\end{aligned}
$$

Determine c_1, c_2, b_1, b_2, a_{11}, a_{12}, a_{21}, and a_{22}.

36 Consider an LP with three $\le$ constraints. The right-hand sides are 10, 15, and 20, respectively. In the optimal tableau, s_2 is a basic variable in the second constraint, which has a right-hand side of 12. Determine the range of values of b_2 for which the current basis remains optimal. (*Hint:* If rhs of Constraint 2 is $15 + \Delta$, this should help in finding the rhs of the optimal tableau.)

37 Use LINDO to solve the Sailco problem of Section 3.10. Then use the output to answer the following questions:

a If month 1 demand decreased to 35 sailboats, what would be the total cost of satisfying the demands during the next four months?

b If the cost of producing a sailboat with regular-time labor during month 1 were \$420, what would be the new optimal solution?

c Suppose a new customer is willing to pay \$425 for a sailboat. If his demand must be met during month 1, should Sailco fill the order? How about if his demand must be met during month 4?

TABLE 66

z	RS	LS	HB	s_1	s_2	s_3	e_4	a_4	rhs
1	0	0	0	0.5	0	0	4.5	$M - 4.5$	4,500
0	0	0	1	0.19	−0.125	0	0.94	−0.94	187.5
0	0	1	0	−0.05	0.10	0	0.75	−0.75	150
0	0	0	0	−0.17	−0.15	1	−1.88	1.88	5,025
0	1	0	0	0	0	0	−1	1	1,000

TABLE 67

z	x_1	x_2	s_2	e_3	a_1	a_3	rhs
1	0	0	−0	0		−1	1,900
0	0	0	−1	1	1	−1	90
0	1	0	−1	0	0	−0	40
0	0	1	−1	0	1	−0	110

TABLE 68

z	x_1	x_2	s_1	s_2	b
1	0	0	2	3	$\frac{5}{2}$
0	1	0	3	2	$\frac{5}{2}$
0	1	1	1	1	1

REFERENCES

The following texts contain extensive discussions of sensitivity analysis and duality:

Bazaraa, M., and J. Jarvis. *Linear Programming and Network Flows.* New York: Wiley, 1990.

Bersitmas, D., and Tsitsiklis, J. *Introduction to Linear Optimization*. Belmont, Mass.: Athena, 1997.

Bradley, S., A. Hax, and T. Magnanti. *Applied Mathematical Programming.* Reading, Mass.: Addison-Wesley, 1977.

Dantzig, G. *Linear Programming and Extensions.* Princeton, N.J.: Princeton University Press, 1963.

Dantzig, G., and Thapa, N. *Linear Programming.* New York: Springer-Verlag, 1997.

Gass, S. *Linear Programming: Methods and Applications,* 5th ed. New York: McGraw-Hill, 1985.

Luenberger, D. *Linear and Nonlinear Programming,* 2d ed. Reading, Mass.: Addison-Wesley, 1984.

Murty, K. *Linear Programming.* New York: Wiley, 1983.

Nash, S., and Sofer, A. *Linear and Nonlinear Programming.* New York: McGraw-Hill, 1995.

Nering, E., and Tucker, A. *Linear Programs and Related Problems.* New York: Academic Press, 1993.

Simmons, D. *Linear Programming for Operations Research.* Englewood Cliffs, N.J.: Prentice Hall, 1972.

Simonnard, M. *Linear Programming.* Englewood Cliffs, N.J.: Prentice Hall, 1966.

Wu, N., and R. Coppins. *Linear Programming and Extensions.* New York: McGraw-Hill, 1981.

The following contains a lucid discussion of DEA:

Callen, J. "Data Envelopment Analysis: Practical Survey and Managerial Accounting Applications," *Journal of Management Accounting Research* 3(1991):35–57.