Chapter 7 Review Solutions

Exercises 1-6, 8-11, 15-16, 23, 25

1. After using the NW corner rule and apply MODI (modified distribution), or the "u-v" method, we get:

	C1	C2	C3	
P 1	75 20	60	69 30	50
P 2	79 10	73 90	68	100
P 3	85 50	76	70	50
Dummy	0	0	0 70	70
Demand	80	90	100	

3. To set this up, think of Jan, Feb and March having both a supply and demand- The supplies being 35, 30 and 35, respectively, and the demands being 30, 30, 20, 20, respectively.

   Here is the optimal solution:

	Jan	Feb	Mar	Dummy	
Jan	400 30	420 5	440	0	35
Feb	425	420 10	440	0 20	30
Mar	420	415 15	410 20	0	35
Demand	30	30	20	20	

5. To set it up, think of the hard drive (HD), the computer memory (CM), and the computer tape (CT) as being the supplies- 200, 100 and 300 each. Then they (computer packets) can be "delivered" as word processing (WP), packaged program (PP), data

file (DF), with “demands” 300, 100 and 100. The costs are costs in time. Notice that Table 68 does need to be modified for our final costs- For example, if WP is accessed 8 times a month, then the time costs for the HD, CM and CT are $8 \times [5, 2, 10]$, or $[40, 16, 80]$. Here is the optimal tableau:

	WP	CM	CT	Dummy	
WP	$\boxed{40}$ 200	$\boxed{16}$ 0	$\boxed{8}$	$\boxed{0}$	200
PP	$\boxed{16}$ 100	$\boxed{4}$	$\boxed{2}$	$\boxed{0}$	100
DF	$\boxed{80}$	$\boxed{32}$ 100	$\boxed{12}$ 100	$\boxed{0}$ 100	300
Demand	300	100	100	100	

8. Notice that we needed a dummy column to proceed. An optimal solution: $x_{11} = 40$, $x_{12} = 10$, $x_{22} = 50$, $x_{32} = 10$, and $x_{34} = 30$.

9. The LP is equivalent to a 2×2 transportation problem. Taking x_1, x_2 for the first row and x_3, x_4 for the second row, the optimal solution is $3, 1, 0, 5$, respectively.

10. You should find that the optimal solution is given by

10	5	
	5	10

$$\Rightarrow \quad z = 130$$

11. (TYPO: s_i should be $s_1 = 16$). Here's the new optimum:

$\boxed{4}$ 10	$\boxed{2}$ 6	$\boxed{4}$	16
$\boxed{12}$	$\boxed{8}$ 4	$\boxed{4}$ 11	15
10	10	11	

$z = 40 + 12 + 32 + 44 = 128$

Shipping 1 unit more cost 2 dollars less.

1 unit switched from X_{22} to X_{12} (savings: \$6)
1 unit more went via X_{23} (cost: \$4)
$\Big\}$ saving \$2

15. Since the change is to the cost for a NBV, changing it will only result in changing that value of $c_{ij} - (u_i + v_j)$, which in this case:

$$(7 + \Delta) - (3 + 2) = 2 + \Delta > 0 \quad \Rightarrow \quad \Delta > -2$$

Therefore, the value of c_{24} can range in the interval $[5, \infty)$

NOTE: It's OK to leave the value of 0 out- This would permit a multiple optimal solution, but the current solution would still be one of them.

16. This changes the cost associated to a basic variable, so we will have to re-work the our u, v computations. Solving the resulting inequalities should yield $-2 \le \Delta \le 2$, so the current basis remains optimal for $11 \le c_{23} \le 15$.

	$v_1 = 6 - \Delta$	$v_2 = 6$	$v_3 = 10$	$v_4 = 2$
$u_1 = 0$	8 $(2+\Delta)$	6 10	10 25	9 (7)
$u_2 = 3+\Delta$	9 45	12 $(3-\Delta)$	$13+\Delta$ 5	7 $(2-\Delta)$
$u_3 = 3$	M $(5+\Delta)$	9 10	16 (3)	5 30

23. Two notes that might help you construct this one- Since customers three has two demands, split it into two columns- one for the regular demand, and one for the extra demand. We will need a dummy plant to make the problem balanced.

Further, we are going to maximize the profit in this problem, not minimize. Therefore, we don't want to use M for the "profit" from the dummy plant to the customer- Rather, we can use $-M$. The tableau is given below- We have given supply and demand in thousands.

	C_1	C_2	C_{31}	C_{32}	C_4	
P_1	65	63	62	62	64	3
P_2	68	67	65	65	62	5
P_3	63	60	59	59	60	5
D	$-M$	$-M$	$-M$	0	0	3
	4	3	3	3	3	