Review Material, Exam 2, Ops Research

The exam will cover the material since Exam 1- Chapter 6 except for 6.6, then all of Chapter 7 (7.1-7.4). **You may write a half-page of note for the exam**, that is, something equivalent to half of a standard 8.5×11 sheet (8.5×5.5 gives 46.75 square inches).

Definitions:

Shadow price, "loop" for the transportation tableau, BFS (for the transportation tableau)

Conversions

- Given a primal, write the dual (and vice-versa).
- Write a transportation problem as a linear program.
- Write the linear program for a transportation problem as a (balanced) transportation tableau, and vice-versa.

Theorems:

Be able to use these theorems to justify your answers. The full theorems are not stated below, but there should be enough there as a memory aid- So you might use these to help you remember which is which. You should look these up to see what the full statements are.

- Chapter 6 (The Dual and Sensitivity)
  - Weak duality: $\mathbf{c}^T\mathbf{x} \le \mathbf{y}^T A\mathbf{x} \le \mathbf{b}^T\mathbf{y}$
  - Strong duality: Optimal solutions iff $\mathbf{c}^T\mathbf{x} = \mathbf{b}^T\mathbf{y}$.
  - Dual Theorem: $\mathbf{y} = (\mathbf{c}_B^T B^{-1})^T$ and $z = w$.
  - Theorem: Shadow prices are the solutions to the dual.
  - Complementary Slackness Theorem: $s_i y_i = 0$ and $e_j x_j = 0$.
- Chapter 7 (Transportation and Related)
  - (Theorem 1) In a balanced transportation problem with m supply points and n demand points, the cells corresponding to a set of $m + n - 1$ variables contain no loop iff the $m + n - 1$ cells yeld a basic soltuion.

Algorithms: Ch 6-7

Be able to compute using these algorithms. In particular, understand when to stop and how to interpret what you have back into the LP. You should also know when you would use each of the algorithms.

- Construction of the Dual- Normal form and not normal.
- The Dual Simplex Method
- Find a BFS to the Transportation Tableau
  - NW Corner Rule
  - Min Cost Rule
  - Vogel's Approx. Method

- MODI, or the $u - v$ method (The Transportation Simplex Method)
  - Determine if a given BFS is optimal (by computing u_i, v_j)
  - If not optimal, find a neighboring BFS that gives a better solution.

Techniques (Ch 6)

- Be able to perform sensitivity analysis via a graphical analysis.
- Understand and use the notation we developed. In particular,

$$\begin{array}{c|c} -\mathbf{c}^T & 0 \\ \hline A & \mathbf{b} \end{array} \quad \longrightarrow \quad \begin{array}{c|c} -\mathbf{c}^T + \mathbf{c}_B^T B^{-1} A & \mathbf{c}_B^T B^{-1}\mathbf{b} \\ \hline B^{-1}A & B^{-1}\mathbf{b} \end{array}$$

- Be able to give the optimal tableau given a current basis. That is, be able to use the linear algebra notation to write down how you would compute the optimal tableau.
- Be able to compute the (final) tableau corresponding to a given basis (directly, without going through the Simplex Method).
- Solve the dual using:
  - Row 0 of the optimal tableau for the primal.
  - Complementary Slackness.
  - the Dual Simplex Method.
- See sensitivity analysis below.

A Summary of Sensitivity Analysis (Ch 6)

The assumptions for sensitivity analysis are that we (i) assume that only one change at a time is made, and (ii) assume we want the current basis to remain optimal. What follows is a summary- See your notes if you want to see more detail.

1. Change the coefficient corresponding to a **non-basic variable** (NBV).
   - Direct analysis without the dual: We just subtract Δ from the optimal Row 0 coefficient, $\hat{c}_j$, to check that the new Row 0 coefficient is non-negative: $\hat{c}_j - \Delta > 0$.
   - Using the dual: Check that the new coefficient still satisfies the j^{th} constraint for the dual:

   $$\mathbf{y}^T \mathbf{a}_j \geq (c_j + \Delta)$$

2. Change the coefficient corresponding to a **basic variable** (BV).

   Remember that the BVs are indexed by the order of the columns of the identity, and there is one per row of the constraint.

   The shortcut formula for the NBVs in Row 0:

   $$\text{Optimal Row } 0 + \Delta \;\text{ Row } i \text{ of optimal tableau} > 0$$

   NOTE: The coefficients for the basic variables are simply set to 0.

3. Change the **right hand side** of a constraint: Optim. RHS $+ \Delta\, i^{\text{th}}$ col. $B^{-1} > 0$
4. Change the **column values** for a non-basic variable. Two ways to compute it:

- We could "price out" the new column values if we are given B^{-1}. If $\mathbf{a}_j$ is the new j^{th} column, then the corresponding column in the optimal tableau is:

$$B^{-1}\mathbf{a}_j$$

  And the new Row 0 value in the j^{th} position is $-c_j + \mathbf{c}_B^T B^{-1}\mathbf{a}_j$

- Given the solution to the dual, $\mathbf{y}$, if we change a column corresponding to a non-basic variable, then we need to be sure that the corresponding constraint in the dual remains satisfied. In the normal case,

$$\mathbf{y}^T\mathbf{a}_j \geq c_j$$

5. Summary using the dual:

   In changing the objective function coefficient of a NBV, changing the column of a NBV, or if we add a new activity (or column), we simply need to determine if the change maintains dual feasibility. If so, the current basis remains optimal. If not, then the current basis is no longer optimal, and we might use the dual simplex method to find the new basis.

Summary of Sensitivity Analysis (Ch 7)

- Change to the objective function coefficient of a NBV:

  As long as the coefficient of x_{ij} in the optimal row 0 is non-negative, the current basis remains optimal.

- Change to the objective function coefficient of a BV:

  This changes the values of the u_i, v_j in the table and they need to be recomputed. That also changes the "Row 0" values (we put those in parentheses), and all these need to be non-negative.

- Change supply s_i and demand d_j by Δ:

  - New z=Old z $+\Delta u_i + \Delta v_j$
  - If x_{ij} corresponds to a BV, just increase x_{ij} by Δ.
  - If x_{ij} corresponds to a NBV, then $x_{ij} = \Delta$. This will automatically create a loop, so we need to remove the loop (Subtract Δ from each loop entry). For the current basis to remain optimal, the new values need to remain non-negative.