

## Example: BFS and Vertices

Original example:

$$\begin{aligned} \min_x \quad & 3x_1 - 4x_2 \\ \text{s.t.} \quad & -2x_1 + x_2 \leq 1 \\ & -x_1 + 2x_2 \geq 5 \\ & x_1 + x_2 \leq 7 \end{aligned}$$

with  $x_1, x_2, x_3 \geq 0$ .

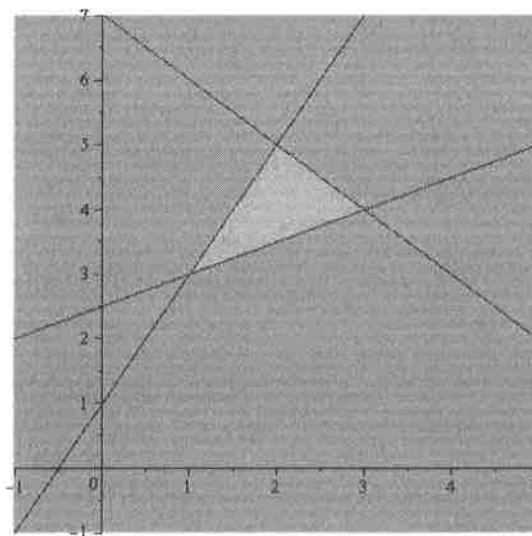
This was translated into standard form:

$$\begin{aligned} \min_x \quad & 3x_1 - 4x_2 + 0s_1 + 0e_2 + 0s_3 \\ \text{s.t.} \quad & -2x_1 + x_2 + s_1 = 1 \\ & -x_1 + 2x_2 - e_2 = 5 \\ & x_1 + x_2 + s_3 = 7 \end{aligned} \Rightarrow \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ e_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$$

A summary of all the basic solutions:

Basic Variables	Solution	Feasible?	Objective
1, 2, 3	3, 4, 3	Yes	-7
1, 2, 4	2, 5, 3	Yes	-14
1, 2, 5	1, 3, 3	Yes	-9
1, 3, 4	7, 15, -12	No	
1, 3, 5	-5, -9, 12	No	
1, 4, 5	-0.5, -4.5, 7.5	No	
2, 3, 4	7, -6, 9	No	
2, 3, 5	2.5, -1.5, 4.5	No	
2, 4, 5	1, -3, 6	No	
3, 4, 5	1, -5, 7	No	

The original feasible set (Can you tell which BFS corresponds to which vertex?)



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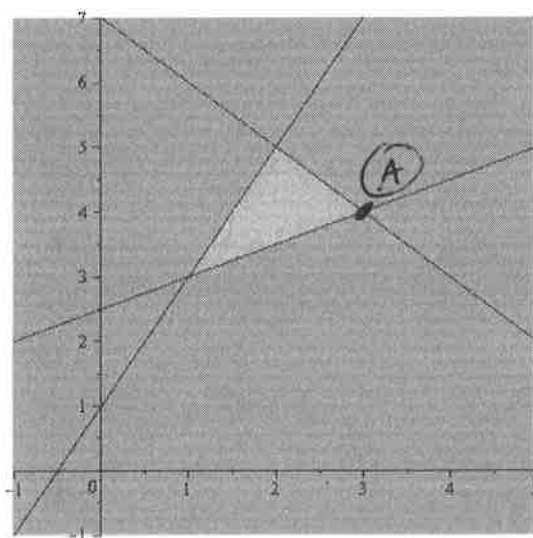
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$x_1 = 3$   
 $x_2 = 4$

$s_1 = 3$   
 $e_2 = 0$   
 $s_3 = 0$

A

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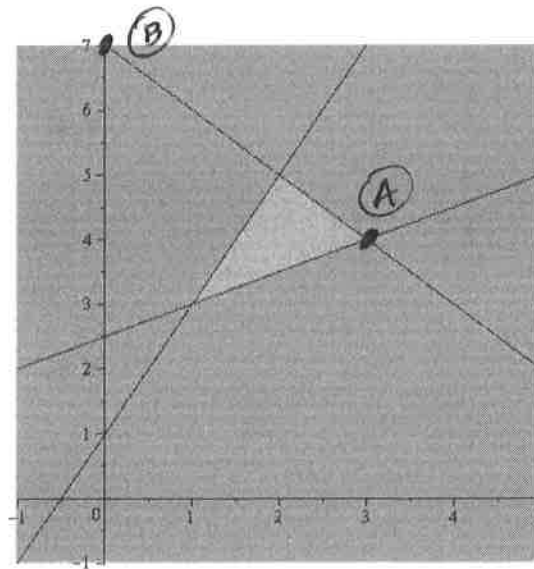
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2, 4, 5	1, -3, 6	No	
3, 4, 5	1, -5, 7	No	

Handwritten notes:

- For (1, 2, 3):  $x_1 = 3, x_2 = 4, s_1 = 3, e_2 = s_3 = 0$  (A)
- For (2, 3, 4):  $x_1 = 0, x_2 = 7, s_1 = -6, e_2 = 9, s_3 = 0$  (B)

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2, 4, 5	1, -3, 6	No	
3, 4, 5	1, -5, 7	No	

$x_1 = 0$   
 $x_2 = 1$   
 $s_1 = 1$   
 $e_2 = -3$   
 $s_3 = 6$

(C)

(D) ← ?

The original feasible set (Can you tell which BFS corresponds to which vertex?)

