

## Example 2: Cycling (Exercise 3, p. 172)

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	-2	-3	1	12	0	0	0
$s_1$	-2	-9	1	9	1	0	0
$s_2$	1/3	1	-1/3	-2	0	1	0

Initially we bring in  $x_2$  as a BV, and replace  $s_2$ .

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	-1	0	0	6	0	3	0
$s_1$	1	0	-2	-9	1	9	0
$x_2$	1/3	1	-1/3	-2	0	1	0

The BFS is now  $(0, 0, 0, 0)$ . Bring in  $x_1$

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	0	0	-2	-3	1	12	0
$x_1$	1	0	-2	-9	1	9	0
$x_2$	0	1	1/3	1	-1/3	-2	0

The BFS is still  $(0, 0, 0, 0)$  even after pivoting. Bring in  $x_4$ :

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	0	3	-1	0	0	6	0
$x_1$	1	9	1	0	-2	-9	0
$x_4$	0	1	1/3	1	-1/3	-2	0

Now bring in  $x_3$ :

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	1	12	0	0	-2	-3	0
$x_3$	1	9	1	0	-2	-9	0
$x_4$	-1/3	-2	0	1	1/3	1	0

Continuing, now  $s_2$  comes back in:

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	0	6	0	3	-1	0	0
$x_3$	-2	-9	1	9	1	0	0
$s_2$	-1/3	-2	0	1	1/3	1	0

And finally  $s_1$  comes back in:

	$x_1$	$x_2$	$x_3$	$x_4$	$s_1$	$s_2$	RHS
	-2	-3	1	12	0	0	0
$s_1$	-2	-9	1	9	1	0	0
$s_2$	1/3	1	-1/3	-2	0	1	0

And we're back where we started!

### Example 3(b): A Closer Look at Degeneracy

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	
	-3	-4	0	0	0	0	0	0	0
$s_1$	-2	1	1	0	0	0	0	0	2
$s_2$	2	-1	0	1	0	0	0	0	4
$s_3$	1	0	0	0	1	0	0	0	3
$s_4$	0	1	0	0	0	1	0	0	4
$s_5$	-1	1	0	0	0	0	1	0	3
$s_6$	1	1	0	0	0	0	0	1	7

$\Rightarrow$  Vertex (non-degen):  $(0, 0)$   
 $\mathbf{s} = [2, 4, 3, 4, 3, 7]^T$

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	
	-11	0	4	0	0	0	0	0	8
$x_2$	-2	1	1	0	0	0	0	0	2
$s_2$	0	0	1	1	0	0	0	0	6
$s_3$	1	0	0	0	1	0	0	0	3
$s_4$	2	0	-1	0	0	1	0	0	2
$s_5$	1	0	-1	0	0	0	1	0	1
$s_6$	3	0	-1	0	0	0	0	1	5

$\Rightarrow$  Vertex (non-degen):  $(0, 2)$   
 $\mathbf{s} = [0, 6, 3, 2, 1, 5]^T$

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	
	0	0	-7	0	0	0	11	0	19
$x_2$	0	1	-1	0	0	0	2	0	4
$s_2$	0	0	1	1	0	0	0	0	6
$s_3$	0	0	1	0	1	0	-1	0	2
$s_4$	0	0	1	0	0	1	-2	0	0
$x_1$	1	0	-1	0	0	0	1	0	1
$s_6$	0	0	2	0	0	0	3	1	2

$\Rightarrow$  Vertex (degen):  $(1, 4)$   
 $\mathbf{s} = [0, 6, 2, 0^*, 0, 2]^T$

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	
	0	0	0	0	0	7	-3	0	19
$x_2$	0	1	0	0	0	1	0	0	4
$s_2$	0	0	0	1	0	-1	2	0	6
$s_3$	0	0	0	0	1	-1	1	0	2
$s_1$	0	0	1	0	0	1	-2	0	0
$x_1$	1	0	0	0	0	1	-1	0	1
$s_6$	0	0	0	0	0	-2	1	1	2

$\Rightarrow$  Vertex (degen):  $(1, 4)$   
 $\mathbf{s} = [0^*, 6, 2, 0, 0, 2]^T$

	$x_1$	$x_2$	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	
	0	0	0	0	0	1	0	3	25
$x_2$	0	1	0	0	0	1	0	0	4
$s_2$	0	0	0	1	0	3	0	-2	2
$s_3$	0	0	0	0	1	1	0	-1	0
$s_1$	0	0	1	0	0	-3	0	2	4
$x_1$	1	0	0	0	0	-1	0	1	3
$s_5$	0	0	0	0	0	-2	1	1	2

$\Rightarrow$  Vertex (degen):  $(3, 4)$   
 $\mathbf{s} = [4, 2, 0^*, 0, 2, 0]^T$

