Homework: After Thursday's Class

If not otherwise stated in the problem, assume that A is $m \times n$ (where A might not be square).

1. Write the following as a mathematical statement (or formula): The projection of $\mathbf{x}$ onto the vector $\mathbf{u}$
2. Show that, if A is an $n \times n$ symmetric matrix, then two eigenvectors from distinct eigenvalues must be orthogonal. (Hint: You might start by considering $\lambda_1 \mathbf{v}_1 \cdot \mathbf{v}_2$)
3. If $\mathbf{u} = [1, 3]^T$, compute the matrix P so that P is an orthogonal projector into the columnspace of $\mathbf{u}$.
4. Show that, if λ is an eigenvalue of $A^T A$, then $\lambda \geq 0$. (Hint: Start with $\|A\mathbf{v}\|^2$)
5. Prove that, if $\mathbf{u}_1, \ldots, \mathbf{u}_k$ are orthonormal, and $\mathbf{x} = c_1\mathbf{u}_1 + \ldots + c_k\mathbf{u}_k$ then $\|\mathbf{x}\|^2 = c_1^2 + \ldots c_k^2$
6. Prove that, if $\mathbf{v}_i$ is orthogonal to $\mathbf{v}_j$, and they are both eigenvectors for $A^T A$, then $A\mathbf{v}_i$ is orthogonal to $A\mathbf{v}_j$.
7. In the previous problem, we did not need to say that $\mathbf{v}_i$ is orthogonal to $\mathbf{v}_j$, because something else should have told us that that was true. What Theorem tells us that the eigenvectors are orthogonal?
8. Problem 27, p. 83 (Matlab Exercise with image of clown)
9. Problems 1, 2 on p. 85 (Generalized Inverses and Matlab)