Research Statement

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I began my career under the thesis supervision of Ken Bogart, where my work was primarily focused on Geometric Representations of Ordered Sets. We looked primarily at Free Triangle Orders, and more generally at (n, i, f)-tube orders, in an attempt to settle the *unit = proper* question for different classes of orders. My work led to a trio of publications with Ken [3],[4][5] and one with a student (Ashifi Gogo) [7], as well as numerous conference presentations.

In Fall 2005, while on sabbatical, I worked with Jean-Paul Doignon at the Universite Libre de Bruxelles, where our work focused on polytopes generated from ordered sets. Our paper combines elements of ordered set theory, matrix theory, and linear programming. Our paper was published in *Order* [6]. This work also introduced me to the rich field of mathematical psychology, and I serve presently as a referee for the Journal of Mathematical Psychology. I use many of the examples involving transitive indifferences and decision-making theory as a side discussion in my calculus classes [1]. In addition, enumerative aspects of this work led me to co-author a paper with Whitman undergraduate Jacob Menashe in the *Journal of Integer Sequences* [11].

In 2007, I teamed with a graduate school colleague (Chris Storm of Adelphi University) to tackle problems relating to the spectrum of hypergraphs. We sought Whitney-type analogs to line graphs of hypergraphs, and to find out when a hypergraph can be reconstructed from its spectrum. This began as a graph theoretic extension of a classic paper by Kac, which asked if one can hear the shape of a drum [15]. Our paper was published in the *Journal of Graph Theory* [12] and led to a presentation at the British Combinatorial Conference in 2009.

In 2008, I was approached by a departmental colleague, Pat Keef, who was working on a problem in Abelian Groups. We collaborated on applications of ordered sets to recognizing when a group is (or is nearly) a direct product of cyclic groups. Though much of the paper rests in Group Theory, it is nice to see a 'non-standard' application of the theory of ordered sets. Our paper appeared in the Italian Journal *Note di Matematica* [10]. This project initialized my interest in problems which span seemingly different branches of mathematics. In particular, I became interested in using the tools of one branch, together with a firm translation, to answer questions in another branch.

In 2010-11 I was fortunate to take a year-long sabbatical. Though I began with the intention of studying Galois Theory as it applied to graphs, I found myself devoting a great deal of time to enumerative problems, and in particular to tiling problems. Specifically, I looked at generalizations of square-domino tilings, which generated sequences beyond the classic Fibonacci counting applications [14]. These problems led to work on the Online Encyclopedia of Integer Sequences. My publication on this work appears in the Journal of Integer Sequences [2]. In particular, I found a link between the 'coordination sequences' in both mathematics and geology (in particular, crystallography).

The work I did with these sequences gave an interesting tie back to my original thesis work on ordered sets. For certain number sequences, the ratio of terms approaches a fixed constant. Most famously, the Golden Ratio ϕ is the limit of the ratio of successive Fibonacci Numbers (with their recursion $a_n = a_{n-1} + a_{n-2}$. My tiling problems led to a lot of work with ψ, the so called 'plastic constant' (which arises from the recursion $a_n = a_{n-2} + a_{n-3}$. This ratio shows up in the work of Richard Padovan and Dom Hans van der Laan, and deals with orders of magnitude and architecture. In applications of partially ordered sets to psychology, one often looks at 'just-noticeable differences' (of sounds, tastes, or other stimuli). Given two stimuli, how 'far apart' do they need to be in order to differentiate between them? The work with the plastic constant leads to a theory of what I've termed 'just-discernable differences.' Given two stimuli, how far apart do they need to be in order to tell how much more intense one is than the other? Not a great deal is written on this topic at present, and I hope to add to the literature in my future endeavors.

My research on number sequences led me to spend a great deal of time with the now-classic book *Proofs that Really Count* by Art Benjamin and J.J. Quinn [13]. In Spring 2013, I was fortunate to work with senior Helen Jenne, a candidate for honors in our department. I gave her the book early on, which is rich with problems and identities in need of combinatorial interpretation. In particular, she latched onto interpretations of continued fractions in terms of tiling, and an unsolved problem involving Euler's continued formula fraction for e. Our work that semester led to looking at derangements (permutations which leave no point fixed) and scramblings (permutations which leave no adjacency fixed). These special permutations arose in my own senior project as an undergraduate. Together with 'stackable' tilings, there was a clean combinatorial interpretation of the continued fraction representation for e.

Our paper was published in the *Journal of Integer Sequences* [9], and has led to several conference presentations.

Near the end of my last sabbatical (Fall 2013), I began work with my departmental colleague David Guichard on a graph constant that we've called the Σ-connectivity of a graph. This measure goes beyond standard vertex connectivity by looking at the removal of subsets of vertices, and determining whether the graph remains connected. Our measure has probabilistic interpretations and gives a better sense of how connected a graph is (under, say, failure of a random set of nodes). Our work in trying to maximize and minimize this measure among graphs led to another application of cographs (series-parallel graphs), and a class of split graphs which we've called *flower graphs*. Our paper has been accepted for publication in the Journal of Combinatorial Mathematics and Combinatorial Computing [8], and has led several conference presentations.

In the coming semesters, I hope to continue to split my time between Enumerative Combinatorics and Graph Theory. I am interested in applying the techniques of bijective and combinatorial recursion proofs to graph enumeration problems, specifically on ordered families and structures of graphs. I've also begun an introductory investigation into using Fibonacci Numbers as a hand-evaluation technique in Bridge (my other passion). As mentioned earlier, I'd like to do more with applications of the 'plastic constant' to perception. I also hope to do more exploration into the uses of (and hindrances from) psychology as pertains to probability and perhaps to mathematics in general. As a population, I find that there are often barriers to using mathematical information to arrive at the best decisions (whether through miscalculations, incomplete information, or misinterpretation of data). In short, I'd like to learn more about how and why this happens. My work thus far has built a wide base from which to draw in the future, and I'm grateful for the opportunities that Whitman has provided me as a teacher-scholar to pursue these goals.

References

[1] B. Balof "Extra Credit" Whitman Magazine, March 2008

[2] B. Balof "Restricted Tilings and Bijections" *Journal of Integer Sequences*, vol 15, article 12.2.3 (2012)

[3] B. Balof and K. Bogart "Simple Inductive Proofs of the Fishburn and Mirkin Theorem and Scott-Suppes Theorem" *Order*, vol 20 p.49-51 (2003)

[4] B. Balof and K. Bogart "Non-Unit Free Triangle Orders" *Order*, vol 23, p.21-35 (2006)

[5] B. Balof and K. Bogart "A Finite Non-Unit Free Triangle Order" *Order*, vol 23, p37-49 (2006)

[6] B. Balof, J.-P. Doignon, S. Fiorini "The Representation Polyhedron of a Semiorder" *Order* Volume 30, Issue 1, pp 103-135 (2013)

[7] B. Balof and A. Gogo "A Non-Unit Free Tetrahedron Order" *Congressus Numerantium*, vol 177, p. 183-200 (2005)

[8] B. Balof and D. Guichard "A New Measure of Graph Connectivity" *Journal of Combinatorial Mathematics and Combinatorial Computing*, accepted for publication (2016)

[9] B. Balof and H. Jenne "Tilings, Continued Fractions, Derangements, Scrambilngs, and e" *Journal of Integer Sequences*, vol 17, article 14.2.7 (2014)

[10] B. Balof and P. Keef "Invariants on primary abelian groups and a problem of Nunke" *Note di Mathematica* vol. 29, no. 2, 83-115 (2009)

[11] B. Balof and J. Menashe "Semiorders and Riordan Numbers" *Journal of Integer Sequences*, vol 10, article 07.7.6 (2007)

[12] B. Balof and C. Storm Constructing isospectral non-isomorphic digraphs from hypergraphs" *Journal of Graph Theory*, v. 63, p. 231-242 (2010)

[13] A. T. Benjamin and J. J. Quinn *Proofs that Really Count: The Art of Combinatorial Proof,* The Mathematical Association of America (2003)

[14] Ralph Grimaldi: "Tilings, Compositions, and Generalizations" *Journal of Integer Sequences*, Vol. 13, Issue 6, article 10.6.5 (2010)

[15] M. Kac "Can One Hear the Shape of a Drum?" *American Mathematical Monthly* v. 73 p 1-23 (1966)