Math 108- First Assignment
Due September 6, 2006

These problems, though mathematical in their thought processes, require explanation to solve. Hence, your answers should be written in prose (preferably typed).

1. You have three boxes of fruit on the table. The boxes are labeled 'Two Apples', 'Two Oranges', and 'One Apple and One Orange'. The labels are correct, but they've each been placed on the wrong box. You are allowed to pull only one piece of fruit from one of the boxes. Can you label the boxes correctly?

2. A king is writing up his will upon learning that his wife is pregnant. He makes the following stipulation: If the queen gives birth to a son, the son is to get $\frac{2}{3}$ of the estate, while the queen gets $\frac{1}{3}$. If the queen gives birth to a daughter, the daughter is to get $\frac{1}{3}$ of the estate, while the queen gets $\frac{2}{3}$. Sure enough, the king dies. The queen, unexpectedly, gives birth to twins, one boy and one girl. What is the fairest division of the king's estate. (As with the taxi problem, there will be multiple answers. Describe at least 2, giving both merits and drawbacks).

3. How many true statements are on this list?

   (a) Exactly Zero of these statements are false.

   (b) Exactly One of these statements is false.

   (c) Exactly Two of these statements are false.

   (d) Exactly Three of these statements are false.

   (e) Exactly Four of these statements are false.

   (f) Exactly Five of these statements are false.

   What if we drop the first statement? What if, instead, we drop word 'Exactly'? What if we do both?

4. In 2004, the Whitman Mathematical Society's T-Shirt read 'The statement on the back of this shirt is true' on the front, and 'The statement on the front of this shirt is False,' on the back. Explain.

5. Alice: Here I have a penny and a quarter. You get to make a statement and if it is true I will give you one of the coins. If it is false, I give you neither coin.

   Bob: That is generous of you. Which coin will you give me if my statement is true?

   Alice: Not telling.

   What statement should Bob make to maximize the amount of money Alice gives him?

   (appeared as Problem of the Week #1034 at Macalester College) Adapted from "Godelian Puzzles," by Raymond Smullyan, in Tribute to a Mathemagican, by Cipra, Demaine, Demaine, and Rogers, AK Peters, 2005.