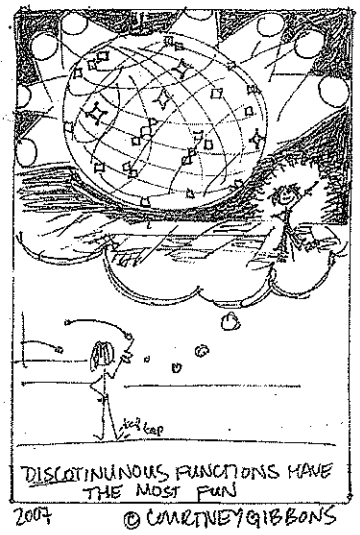
Math 125: Exam the First KEY

October 7, 2011

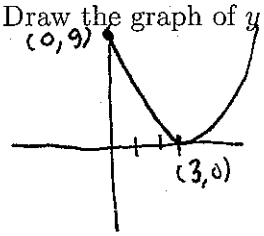
This exam is closed book, closed notes, closed colleague. You have until 9:55 to finish. **READ ALL INSTRUCTIONS CAREFULLY.** Please read the statement below and sign it when you are finished.

I have not used my calculator on this examination except for arithmetic, trigonometric, logarithmic, and exponential functions. I certify that the work on this exam is my own and that I have not discussed the specific contents of this exam with anyone prior to my taking it.

Signature:

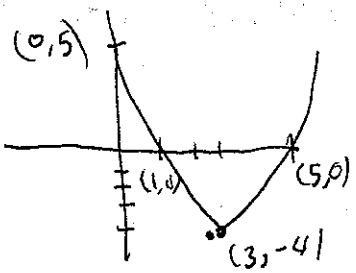


1. (a) Draw the graph of $y = (x-3)^2$, labelling both $x-$ and $y-$intercepts.



(15)

(b) Draw the grpah of $y = (x-3)^2 - 4$, labelling both $x-$ and $y-$intercepts.



$(x-3)^2 - 4 = 0$

$(x-3) = \pm 2$

$x = 3 \pm 2 = 1, 5$

2. (a) Given a function $f(x)$, state the definition of the derivative, $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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(b) Use the definition to calculate the derivative of $f(x) = \frac{1}{1-x}$.

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{1-(x+h)} - \frac{1}{1-x}}{h} = \lim_{h \to 0} \frac{(1-x) - (1-x+h)}{(1-(x+h))(1-x)} = \lim_{h \to 0} \frac{+h}{(1-(x+h))(1-x)\,h}$$

$$= \lim_{h \to 0} \frac{+1}{(1-(x+h))(1-x)} = \frac{+1}{(1-x)^2}$$

3. Calculate the following limits, using the methods indicated.

(a) Numerically:

$$\lim_{x \to 0} \frac{(\cos x) - 1}{x^2} = f(x)$$

x	f(x)

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x^2} = -.5$$

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(b) Algebraically:

$$\lim_{x \to 1} \frac{x^3 - x}{x^2 + 4x - 5}$$

$$= \lim_{x \to 1} \frac{(x)(x^2-1)}{(x+1)(x+5)} = \lim_{x \to 1} \frac{(x)(x-1)(x+1)}{(x-1)(x+5)} = \lim_{x \to 1} \frac{x(x+1)}{(x+5)} = \frac{2}{6} = \frac{1}{3}$$

(c) Using a Theorem:

$$\lim_{x\to 0} x^2 \cos\left(\frac{1}{x^2}\right)$$

$-1 \le \cos\left(\frac{1}{x^2}\right) \le 1$, so $(-x^2 \le x^2 \cos\frac{1}{x^2} \le x^2)$ so by the Squeeze Theorem since $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$, $\lim_{x\to 0} x^2\cos\left(\frac{1}{x^2}\right) = 0$.

(d) Algebraically: $\infty - \infty$

$$\lim_{x\to\infty} (\sqrt{x^2 - 6x + 8} - x)$$

$\lim_{x\to\infty} \sqrt{x^2-6x+8} - x \left(\frac{\sqrt{x^2-6x+8}+x}{\sqrt{x^2-6x+8}+x}\right) = \lim_{x\to\infty} \frac{x^2-6x+8-x^2}{\sqrt{x^2-6x+8}+x} = \lim_{x\to\infty} \frac{-6x+8}{\sqrt{x^2-6x+8}+x}$

Compare coeff's $\frac{-6}{2} = -3$.

4. Economists sometimes use the *rule of 70* to calculate how long it will take for an investment to double. If money is invested at r% per year, then it will take $\frac{70}{r}$ years for that investment to double in value.

(a) How long does it take an investment at a rate of 2% to double?

2% interest → doubling time = $\frac{70}{2} = 35$ years

(b) How long does it take an investment at a rate of 5% to double?

5% interest → doubling time = $\frac{70}{5} = 14$ years

(15)

(c) If we invest \$10000 at 2% per year, what will it be worth after 35 years?

By our doubling rule, it will exactly double → \$20,000

(d) If we invest \$10000 at 5% per year, what will it be worth after 35 years?

B = balance

$B(t) = 10{,}000\,(2)^{t/14}$

$B(35) = 10{,}000\,(2)^{35/14} = 10{,}000 \cdot 2^{2.5}$

$= \$40{,}000\sqrt{2}$

5. Find the equation of the tangent line to $f(x) = \frac{2x^2+1}{\sqrt{x}}$ at $x = 9$. (You may use the derivative shortcuts here)

$f(x) = \frac{2x^2}{\sqrt{x}} + \frac{1}{\sqrt{x}} = 2x^{3/2} + x^{-1/2}$

$f(9) = \frac{163}{3}$ $\quad f'(x) = 3x^{1/2} - \frac{1}{2}x^{-3/2}$

$f'(3) = 3\sqrt{9} - \frac{1}{2(\sqrt{9})^3} = 9 - \frac{1}{54} = \frac{485}{54}$

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Tangent line: $y - \frac{163}{3} = \frac{485}{54}(x - 9)$

6. An object is fired off of a cliff on Planet X. Its height, as a funciton of time, is given by $h(t) = 2.1 + 1.8t - 0.3t^2$. ($h$ is measured in meters, t is measured in seconds[1])

(a) At what time does the object hit the ground, and how fast is it going when it does? (You may use the derivative shortcuts here).

When is $h = 0$?

$2.1 + 1.8t - 0.3t^2 = 0$

easier: $-0.3(t^2 - 6t + 7) = 0$

$-0.3(t-7)(t+1) = 0$

$t = 7$ sec.

$v(t) = 1.8 - 0.6t$

$v(7) = 1.8 - 4.2 = -2.4$ m/s

speed $= 2.4$ m/s.

(b) What is the acceleration due to gravity on Planet X?

acceleration $=$

$h''(t) = -0.6$ m/s^2

(15)

(c) Is Planet X larger or smaller than Earth? Explain.

Planet x is smaller, as it has a lower acceleration due to gravity.

[1]Pay attention to your units in your answers here.