Practice Test 2

1. Find $f'(x)$ in each case:
   (a) $f(x) = x^3 + 6x + 7$
   (b) $f(x) = (3x + 1)(x^2 + 1)^2$
   (c) $f(x) = \sqrt{x^3 - x^4}$
   (d) $f(x) = \dfrac{\ln(x)}{x}$
   (e) $f(x) = \cos(x^2)$
   (f) $f(x) = \ln(\sin(x) + 2)$.
   (g) $f(x) = (2x)^x$
   (h) $f(x) = \arcsin(2x + 1)$
   (i) $f(x) = \arcsin(\cos(x))$. In this case, simplify your answer as much as possible.
2. Find $\frac{dy}{dx}$ using implicit differentiation:
   (a) $y^5 + x^2y^3 = 1 + x^4y$
   (b) $y\sec(x) = x\tan(y)$
   (c) $e^{xy} = 1$
3. Find a formula for $\cos(2x)$ by using the fact that $\sin(2x) = 2\sin(x)\cos(x)$ and differentiating both sides.
4. Find the equation of the tangent line to $f(x) = \sin(x)\cos(x)$ at $x = \frac{\pi}{4}$
5. What does it mean for two curves to be perpendicular at their points of intersection? Prove that the circle $x^2 + y^2 = 1$ and the line $y = x$ are perpendicular.
6. Suppose that $x^2 + xy + y^2 = 1$. Determine where this curve has horizontal and vertical tangents.
7. Suppose that a population is declining at a rate of 3% per year. If there are 5,000 people now, how soon will the population go below 3,500?
8. Describe the differences between interests compounding *yearly*, *monthly*, and *continuously*. Which gives us more return on our investment and why? Suppose that we invest \$50,000 at a rate of 7% for 30 years. What are the differences in the balance at the end of that time if interest is compounded yearly versus monthly versus continuously?
9. My morning oatmeal is heated to a temperature of $68°C$. Five minutes later, it is at $58°C$. The temperature in the cold kitchen is $18\ °C$. How soon will it be $48\ °C$, the temperature at which I can eat it?
10. A rocket takes off the ground at a rate of 250 miles/hour. We are observing the rocket from a point on the ground 5 miles away. How fast is our distance to the rocket changing 2 minutes after liftoff?
11. Approximate cos(3.1515926535....).
12. Find the tangent line to $f(x) = x^3 + 6x + 1$ at $x = 2$ and use it to approximate $f(1.99)$.