Prac Exam 2

1

a. $f(x) = x^3 + 6x + 7$

$f'(x) = 3x^2 + 6$

c. $f(x) = \sqrt{x^3 - x^4} = (x^3 - x^4)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(x^3 - x^4)^{-\frac{1}{2}}(3x^2 - 4x^3)$

e $f(x) = \cos(x^2)$

$f'(x) = (-\sin(x^2))\,2x$

g $f(x) = (2x)^x$

$y = (2x)^x$

$\ln y = \ln(2x)^x = x \ln(2x)$

$\frac{1}{y}\left(\frac{dy}{dx}\right) = \ln(2x) + x\,\frac{1}{2}\,2$

$\left(\frac{dy}{dx}\right) = (\ln(2x)+1)\,y^{2x}$

$\frac{dy}{dx} = (\ln(2x)+1)\,(2x)^x$

I $f(x) = \arcsin(\cos(x))$

$f'(x) = \frac{1}{\sqrt{1-\cos^2 x}} \cdot (-\sin x)$

$f'(x) = \frac{1}{\sqrt{\sin^2 x}} \cdot -(\sin x) = -1$

2

b. $y \sec x = x \tan y$

$\frac{dy}{dx}\sec x + y(\sec x \tan x) = \tan y + x \sec^2 y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\tan y - y \sec x \tan x}{\sec x - x \sec^2 y}$

c. $e^{xy} = 1$

$e^{xy}\left(y + x\frac{dy}{dx}\right) = 0$

$y + x\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{-y \cdot e^{xy}}{x \cdot e^{xy}}$

3. $\sin(2x) = 2\sin x \cos x$

$2\cos 2x = 2\sin x(-\sin x) + 2\cos x(\cos x)$

$2\cos 2x = 2\cos^2 x - 2\sin^2 x$

$\cos 2x = \cos^2 x - \sin^2 x$

5 Tangent lines are perpendicular

$y = x$ $\quad x^2 + y^2 = 1$

↑ ↑

Slope = 1 $\quad$ slope $= \frac{dy}{dx} = 2x + 2y\left(\frac{dy}{dx}\right) = 0$

$= \frac{-x}{y} = \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-x}{y} = -1$

Negative reciprocals

7 $\frac{dP}{dt} = -.03P$ $\quad P = Ce^{-.03t}$

$P(0) = 5000 = Ce^{-.03(0)}$

$P(t) = 5000e^{-.03t} = 3500$ $\quad P(0) = 5000$

$= e^{-.03t} = .7$

$t = \frac{\ln .7}{.03} = 11.88$ years

9. $T_0 = 68°$ $\quad T = T_a + Tde^{kt}$

$T = 18 + 50e^{kt}$

$T(5) = 18 + 50e^{k \cdot 5} = 58$

$k = -.0446$

$T(?) = 18 + 50e^{-.0446t} = 48$

$= 11.46$ mins

Prac Exam 2 KEY

1)

b. $f'(x) = (3x+1)\left[2(x^2+1)\cdot 2x\right] + (x^2+1)^2 \cdot 3$

d. $f'(x) = \dfrac{1/\ln x}{x^2}$

f. $f'(x) = \dfrac{1}{\sin x + 2} \cdot \cos x$

h. $f(x) = \arcsin(2x+1)$ $\quad \frac{d}{dx}(\arcsin(x)) = \dfrac{1}{\sqrt{1-x^2}}$

$f'(x) = \dfrac{1}{\sqrt{1-(2x+1)^2}} \cdot 2$

i) $f(x) = \arcsin(\cos(x))$

$f'(x) = \dfrac{1}{\sqrt{1-\cos^2 x}}(-\sin x)$

$= \dfrac{1}{\sqrt{\sin^2 x}} \cdot (-\sin x) = -1$

2) a. $\dfrac{dy}{dx} = \dfrac{4x^3y - 2xy^3}{5y^4 + 3x^2y^2 - x^4}$

c. $\dfrac{dy}{dx} = \dfrac{-y}{x}$

12) $y = 21 + 18(x-2)$

$f(1.99) \approx 21 + 18(1.99-2)$

20.82

4) $y - \frac{1}{2} = 0\left(x - \frac{\pi}{4}\right)$

$y = \frac{1}{2}$

6) vertical tangents $y = \pm\sqrt{1/3}$ $\quad x = \pm 2\sqrt{1/3}$

8) yearly $= 50{,}000(1.07)^{30} = 380{,}612$

monthly $= 50{,}000\left(1 + \frac{.07}{12}\right)^{360} = 405{,}825$

continuously $= 50{,}000\, e^{.07(30)} = 408{,}308$

10) $\dfrac{dz}{dt} = \dfrac{(50/6)(250)}{\sqrt{(50/6)^2 + 5^2}}$ mi/hr