Math 125: Practice Final

These practice problems are largely comprised of problems from the Fall 2007 Math 125 Final Exam. They are meant to represent the types of problems that you're likely to encounter on the final. Note that this set of problems is not exhaustive. Any inclusions or omissions here should not induce you to study more or fewer topics.

1. (a) Given a function $f(x)$, state the definition of the derivative, $f'(x)$.
   (b) Use the definition to calculate the derivative of $f(x) = \frac{1}{x^2+3}$
2. Find the first and second derivatives of $f(x) = (x^2+1)^{\frac{3}{2}}$.
3. Find the equation of the tangent line to $f(x) = \ln(x^3+1)$ at the point where $x = 2$.
4. Calculate
$$\lim_{x\to\infty} \frac{2x}{\sqrt{3x^2+1}}$$
5. Consider the hyperbolas $xy = c$ and $x^2 - y^2 = k$. Explain why any curve of the first type must be perpendicular to any curve of the second type.
6. Draw a curve $f(x)$ that is bounded for all x, increasing and concave up for $x < a$, and decreasing and concave up for $x > a$. Can such a curve be differentiable at $x = a$? Explain.
7. Find (and classify) all critical points and all points of inflection of $f(x) = x^4 - 6x^2 + 8$.
8. (a) Find the value of x that minimizes the function $f(x) = (x-a)^2 + (x-b)^2$ (a and b are constants).
   (b) Find the value of x that minimizes $f(x) = (x-a)^2 + (x-b)^2 + (x-c)^2$ (a, b, and c are constants).
   (c) Generalize to find the value of x that minimizes $f(x) = (x-a_1)^2+(x-a_2)^2+\ldots+(x-a_k)^2$ (all a_i are constants).
9. The volume of a cube in increasing at a rate of 10 cm^3/min.
   (a) How fast is the length of a side increasing when that length is 30 centimeters?
   (b) How fast is the surface area changing at this time?
10. Suppose that $f(x) = x^3 + 3x + 1$ Show that f has a root between $x = -1$ and $x = 0$.
11. Let $f(x) = \sqrt[3]{x}$
    (a) Use $f(x)$ and its tangent lines to approximate both $\sqrt[3]{1.02}$ and $\sqrt[3]{8.02}$
    (b) Check your results on your calculator. Which estimate is closer (Give the difference to as many decimal places as your calculator can handle)? What explains this?
12. Let $f(x) = x + \frac{1}{x^2}$
    (a) Find the point c guaranteed by the Mean Value Theorem such that $f'(c) = \frac{f(2)-f(1)}{2-1}$.
    (b) Is such a point guaranteed for $f(x)$ on the interval $[-2, 2]$? Explain.