

# KEY

## Math 125-Quiz 21

November 2, 2011

The population of a town is growing at a rate of 8% per year. The town is losing a total of 480 people per year via emigration.

1. Express the above as a differential equation (a derivative of population with respect to time).

$$\frac{dP}{dt} = .08P - 480$$

2. What is the equilibrium solution of this population? (That is, when is growth zero?)

$$\begin{aligned} \frac{dP}{dt} = 0 &\Rightarrow .08P - 480 = 0 && .08P = 480 \\ &&& P = \frac{480}{.08} = 6,000 \end{aligned}$$

3. Show that, if the initial population of the town is 10,000, then

$$P(t) = 6000 + 4000e^{.08t}$$

is a solution to the differential equation. Do this by

- (a) determining  $P(0)$ ,

$$P(0) = 6000 + 4000e^{.08(0)} = 6000 + 4000 = 10,000$$

- (b) then differentiating  $P(t)$  and rewriting the derivative in terms of  $P$ .

$$\begin{aligned} \frac{dP}{dt} &= 4000(e^{.08t}) \cdot .08 \\ &= .08(P - 6000) = .08P - 480 \quad \checkmark \end{aligned}$$

4. If the initial population of the town were 5,000 instead, give an equation for  $P$ .

$$P(0) = 5,000 \rightarrow$$

$$P = 6000 - 1000e^{.08t}$$

5. For each scenario (initial population 10,000 and 5,000), what is the long-term projected population?

$$\text{If } P_0 = 10,000 \quad \text{Population} \rightarrow \infty$$

$$\text{If } P_0 = 5,000 \quad P \rightarrow 0$$