KEY

Math 125-Quiz 22
November 4, 2011

This quiz is due Monday at 8 AM. You may discuss the quiz with one another, and use your book and your notes. However, the work that you turn in should be your own, that is, no copying directly from or for your colleagues.

1. Prof. B^2 needs his coffee before his 9AM Calculus class. When he pours the coffee at 8:30 AM, it is $95°C$. Ten minutes later, it has cooled to $75°C$. In order for him to be able to drink it, it must be no more than $50°C$. If the temperature in his office is $20°C$, will he be able to drink it before class?

$T = T_a + T_d e^{kt}$

$T_a = 20°$

$T_d = 95 - 20° = 75°$

$T = 20 + 75e^{kt}$

$75 = 20 + 75e^{k(10)}$

$\frac{55}{75} = e^{k \cdot 10}$ $\quad k = \frac{\ln \frac{55}{75}}{10} = -.03.1015$

Can he drink the coffee?

$T = 50 = 20 + 75e^{-.031015t}$

$\frac{30}{95} = e^{-.031015t}$

$t = \frac{\ln \frac{30}{75}}{-.031015} = 29.54$ minutes

So yes! (But Barely!)

2. A certain town has 1,600 new people move in each year, but at the same time, loses 4% of its population.

   (a) Express the above as a differential equation (a derivative of population with respect to time).

   $\frac{dP}{dt} = 1600 - .04P$

   (b) What is the equilibrium solution of this population? (That is, when is growth zero?)

   $1600 - .04P = 0$

   $P = \frac{1600}{.04} = 40,000$

(c) Show that, if the initial population of the town is 50,000, then

$$P(t) = 40000 + 10000e^{-.04t}$$

is a solution to the differential equation. Do this by

i. determining $P(0)$,

$P(0) = 40000 + 10000\, e^0 = 50{,}000$

ii. determining $\lim_{t \to \infty} P(t)$

$\lim_{t \to \infty} 40000 + 10000\, e^{-.04t} \Rightarrow \lim = 40{,}000$

↳ 0 for large t

iii. then differentiating $P(t)$ and rewriting the derivative in terms of P.

$\frac{dP}{dt} = 0 + 10{,}000\, e^{-.04t}(-.04)$

$= -.04(P - 40{,}000)$

$= 1600 - .04P$ ✓ check.

(d) If the initial population of the town were 20,000 instead, give an equation for P.

$P(t) = 40000 - 20000\, e^{-.04t}$

(e) If the initial population were 20,000, would the population still tend towards the equilibrium solution in the long run?

Yes, as the second term would still drop off!