KEY

Math 125-Quiz 28 (Final Quiz)
December 7, 2011

This quiz is due Friday at 9AM. You may use your book and your notes. You are free to collaborate with colleagues, but the work that you turn in should be your own (ie, no copying from one another). Be careful about justifying your answers and showing your work.
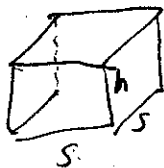
1. Find two positive numbers whose product is 100 and whose sum is as small as possible.

$$x+y=100$$
$$P=xy \quad \leftarrow \text{minimize}$$
$$P=x(100-x)$$
$$P'=100-2x=0$$
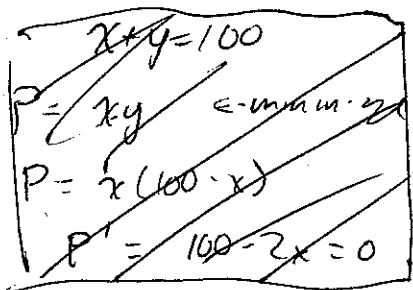
$$xy=100$$
$$S=x+y \quad \leftarrow \text{minimize}$$
$$S=x+\frac{100}{x}$$
$$S'=1-\frac{100}{x^2}=0$$

$$\frac{100}{x^2}=1$$

This is a minimum, as
$$S''=\frac{200}{x^3}>0$$
$$x=10$$
$$y=\frac{100}{10}=10$$

2. A box with a square bottom and top is to be made with 96 in.^{2} of material. Find the dimensions that maximize the volume.

$$V=s^2h$$

$$96=2s^2+4sh$$
$$h=\frac{48-s^2}{2s}$$

$$V=s^2\left(\frac{48-s^2}{2s}\right)=24s-\frac{s^3}{2}=0$$
$$V'=24-\frac{3s^2}{2}=0$$

$$s^2=16$$
$$s=4$$
$$h=\frac{48-16}{4\cdot2}=\frac{32}{8}=4$$

3. A *Norman Window* is made by placing a semicircle atop a rectangle, that is, the diameter of the semicircle is the width of the rectangle. If the perimeter of such a window is to be 30 inches, what dimensions give the largest area?

$$P=2r+2h+\pi r=30 \rightarrow h=\frac{30-2r-\pi r}{2}$$
$$A=2rh+\frac{1}{2}\pi r^2 \qquad =15-\left(\frac{2+\pi}{2}\right)r$$

$$A=2r\left(15-\left(\frac{2+\pi}{2}\right)r\right)+\frac{1}{2}\pi r^2$$
$$A=30r-(2+\pi)r^2+\frac{1}{2}\pi r^2$$
$$A'=30-2(2+\pi)r+\pi r$$
$$=30-(4+\pi)r=0$$
$$r=\frac{30}{4+\pi}$$

$$h=15-\left(\frac{2+\pi}{2}\right)\left(\frac{30}{4+\pi}\right)$$
$$15-15\left(\frac{2+\pi}{4+\pi}\right)$$
$$15\left(1-\frac{2+\pi}{4+\pi}\right)$$
$$h=\frac{30}{4+\pi}$$