

Final Examination: Math 125

You have two hours to complete this closed-book-closed-notes-closed-colleague examination. Please sign the following statement when finished.

I have not used my calculator on this examination except for basic arithmetic, trigonometry, logarithmic, and exponential functions. I have not discussed the contents of this exam with anyone prior to my taking it. Signature:

1. (15 points) State the definition of the derivative. Use it to calculate the derivative of $f(x) = \sqrt{x^2 - 1}$

2. (15 points) A box with a square bottom is to have volume 1000 in^3 . Find the dimensions that minimize the surface area. What shape is the box? Does this shape depend on the volume? (ie, if there were a different required volume, would the box still have this shape?)
3. (10 points) A cannon ball is dropped off of a 361 foot cliff. A second cannonball is fired off of a 240 foot cliff with an initial upward velocity of 2 feet/second. Which hits the ground first? Which hits the ground traveling faster?

4. (10 points) Find the equations of all tangent lines to $y = x^2 + 8x + 7$ that go through the point $(3, 4)$.

5. (10 points) Calculate $\lim_{x \rightarrow 0} \frac{\sin(4x) - 3x}{2x}$

6. (30 points, **READ CAREFULLY**) Consider $f(x) = 1 - 3x + 5x^2 - x^3$. Find all intervals where the graph is increasing, decreasing, and classify all critical points. Find all intervals where the graph is concave up, concave down, and any inflection points. Find the y-intercept (through elementary math), and the x intercept (via Newton's method, to 4 decimal places, its around 4.5). Plot the critical and inflection points and intercepts and draw a rough sketch of $f(x)$.

7. (15 points) The formula for the temperature (in degrees F) of a corpse is given by

$$H(t) = 68 + 30.6e^{-.4t}$$

A woman claims ‘The man was dead when I got here, four hours ago.’ If the body temperature is 77 degrees Fahrenheit, is the woman correct? If you determine she is wrong, figure out when the murder occurred.

What is the temperature in the room?

Determine an equation for the rate of change of temperature with respect to time. Express this equation in terms of $H(t)$.

8. Suppose that $xy^2 - x^2y = 2$.

(a) (10 points) Find $\frac{dy}{dx}$, and find the equation of the tangent line to the curve at the point $(1, 2)$.

(b) (5 points) For any y value, how many possible x values are there?

9. (15 points) A spherical balloon is leaking air at the rate of $3 \text{ in}^3/\text{min}$. How fast is the surface area changing when there are 30 in^3 of air in the balloon? (Hint: first find out how fast the radius is changing).

10. (15 points) Determine $\lim_{x \rightarrow 2} \frac{3}{x-1}$. Find a suitable δ for $\epsilon = .02$