Practice Exam 1

1. (a) Suppose that $f(x) = \frac{1}{x}$. Estimate the area under the curve between $x = 1$ and $x = 3$ using right-hand endpoints and 4 subdivisions.

   (b) Give the formula for R_n (in sigma notation). This is the formula used to estimate the area under the curve using right-hand endpoints and n subdivisions.

   (c) What is $\lim_{n\to\infty} R_n$? Find both an expression and an exact value (Note: I'm not looking for heavy algebraic manipulations here, just a statement using calculus.)

2. Find the following antiderivatives:

   (a) $\int 3x^3 + \cos(x)dx$

   (b) $\int_0^{\pi} \frac{\sec^2(x)}{1+\tan^2(x)}dx$

   (c) $\int_{\ln 2}^{\ln 3} e^x e^{e^x} dx$

3. Find the ratio of area above the x axis and area below the x axis bound by the curve $y = x^3 - 5x^2 - 17x + 21$.

4. Suppose that you know that $\int_{-a}^{a} x^3 g(x)dx = 0$ for all a. What kind of function is $g(x)$? Explain.

5. Find the volume of the solid generated by revolving the area under the curve $y = \sqrt{\sin(x)}$ between 0 and π about the x-axis.

6. Describe the solid whose volume is calculated in each of these integrals.

   (a) $\int_0^1 \pi e^{-2x} dx$

   (b) $\int_0^1 2\pi(1-x)(e^{-x^2})dx$

   (c) $\int_0^1 \pi(e^{2x} - e^{-2x})dx$

7. A conical tank has radius 10 meters and depth 30 meters. Suppose the tank is filled with water to a depth of 25 meters. Find the work done in emptying the tank by pumping the water to the top of the tank. How would this change if the tank were a pyramid with a square top of side length 10 meters instead of a cone?