1. (a) Suppose that
$$f(x) = \int_2^x \cot(t^3) + 2t \; dt$$

Find $f'(x)$.

(b) Find $f'(x)$ if
$$f(x) = \int_2^{x^2} \cot(t^3) + 2t \; dt$$

2. (a) Find
$$\int_3^{10} \frac{x}{x^2 - 4} \; dx$$

(b) Discuss
$$\int_{-1}^{1} \frac{x}{x^2 - 4} \; dx$$

(c) Discuss
$$\int_0^5 \frac{x}{x^2 - 4} \; dx$$

(d) Discuss
$$\int_4^{\infty} \frac{x}{x^2 - 4} \; dx$$

(e) Discuss
$$\int_4^{\infty} \frac{x}{(x^2 - 4)^2} \; dx$$

3. Consider the area under the curve $y = xe^x$ for $0 \le x \le 1$.

(a) Set up the integral that gives this area.

(b) Set up the integral that gives the volume when this area is revolved around the x-axis.

(c) Set up the integral that gives the volume when this area is revolved around the y-axis.

(d) Set up the integral that gives the volume when this area is revolved around the line $x = 1$.

(e) Set up the integral that gives the volume when this area is revolved around the line $y = -2$.

4. Determine the following integrals

(a)
$$\int (x^2 + 1)e^{-x} \; dx$$

1

(b)
$$\int \cos^2(x) \tan^3(x) \, dx$$

(c)
$$\int \frac{t^5}{\sqrt{t^2+1}} \, dt$$

(d)
$$\int \frac{x-6}{x^2+4x+3} \, dx$$

(e)
$$\int \frac{\sqrt{x-4}}{x} \, dx$$

(f)
$$\int \frac{\arctan \sqrt{x}}{\sqrt{x}} \, dx$$

5. Set up the integral to compute the length of one period of the curve $y = \sin x$. Also, set up the integral to compute the surface area of the solid generated by revolving this curve about the x-axis.

6. Define a sequence $\{a_n\}_{n=1}^{\infty}$ by $a_1 = 1$ and $a_n = a_{n-1}^2 - 1$. What are the first six terms of the sequence? Does the sequence approach a limit? If so what? If we define $b_n = a_n^n$, does the series $\sum_{n=1}^{\infty} b_n$ converge?

7. Determine the convergence or divergence of the following series.

(a)
$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{2 - n^3}$$

(b)
$$\sum_{n=1}^{\infty} \ln\left(\frac{2n}{n-3}\right)$$

(c)
$$\sum_{n=1}^{\infty} \frac{n^{2n}}{(1 + 2n^2)^n}$$

(d)
$$\sum_{n=1}^{\infty} \frac{(-1)^n n + 1}{2n^2 + 1}$$

(e)
$$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$

(f)
$$\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5^n}$$

8. Determine
$$\int \frac{e^x}{x}\, dx \text{ and } \int \frac{e^{-x}}{x}\, dx$$

by using series.

9. Determine c so that
$$f(x) = \begin{cases} \frac{c}{x^2} & x > 2 \\ 0 & x < 2 \end{cases}$$

is a probability density function.

10. Find the center of mass of a plate in the shape of the area under the curve $y = \sin 2x$ of density ρ, between $x = 0$ and $x = \frac{\pi}{2}$.

11. Snow is falling on the ground at the rate of 4 inches/minute. It is melting at a rate of 75% How much snow is on the ground after 5 hours? How much snow remains on the ground if it continues to snow indefinitely?