Math 126: Practice Final Exam
Fall 2012

1. (a) Suppose that

$$f(x) = \int_2^x \cot(t^3) + 2t\, dt$$

Find $f'(x)$.

(b) Find $f'(x)$ if

$$f(x) = \int_2^{x^2} \cot(t^3) + 2t\, dt$$

2. Consider the area bound by the y-axis, the line $y = 5$, and the curve $y = x^2 + 1$.

   (a) Set up integrals to find this area using both horizontal and vertical divisions. Solve both integrals and check that your answers are the same.

   (b) Set up, but do not compute, the integrals for the volumes obtained by revolving this area about the y-axis and about the x-axis. You may do this using whatever method you feel is easiest.

3. Solve the following antiderivatives

   (a)

$$\int \frac{\arctan \sqrt{x}}{\sqrt{x}}\, dx$$

   (b)

$$\int x^5 \cos x^3\, dx$$

   (c)

$$\int (x+1)\cos(x^2 + 2x)\, dx$$

   (d)

$$\int \frac{2x}{x^2 + 3x - 10}\, dx$$

   (e)

$$\int \frac{t^2 - 1}{t^2 + 1}\, dt$$

(f)

$$\int \sec^4(x)\tan^4(x)\,dx$$

4. Find the following antiderivatives

   (a)

   $$\int \cos^2(2x)\sin(2x)\,dx$$

   (b)

   $$\int \ln(x^2)\,dx$$

   (c)

   $$\int \frac{5}{x^2+x-6}$$

5. Let $f(x) = \sqrt{1-x}$.

   (a) Find the area bound by $f(x)$ and the x and y-axes.

   (b) Revolve this area about the line $x = 1$. Find the volume using

      i. Cylindrical Shells

      ii. Washers

6. Determine if

   $$\int_2^\infty \frac{dx}{\sqrt{x}(1+x)}$$

   is convergent or divergent. If you decide it's convergent, to what does it converge? (Hint: Look for a substitution).

7. (a) Find

   $$\int_3^{10} \frac{x}{x^2-4}\,dx$$

   (b) Discuss

   $$\int_{-1}^{1} \frac{x}{x^2-4}\,dx$$

   (c) Discuss

   $$\int_0^5 \frac{x}{x^2-4}\,dx$$

(d) Discuss

$$\int_4^\infty \frac{x}{x^2-4}\,dx$$

(e) Discuss

$$\int_4^\infty \frac{x}{(x^2-4)^2}\,dx$$

8. Consider the area under the curve $y = xe^x$ for $0 \le x \le 1$.

   (a) Set up the integral that gives this area.

   (b) Set up the integral that gives the volume when this area is revolved around the x-axis.

   (c) Set up the integral that gives the volume when this area is revolved around the y-axis.

   (d) Set up the integral that gives the volume when this area is revolved around the line $x = 1$.

   (e) Set up the integral that gives the volume when this area is revolved around the line $y = -2$.

9. Set up the integral to compute the length of one period of the curve $y = \sin x$. Also, set up the integral to compute the surface area of the solid generated by revolving this curve about the x-axis.

10. Determine c so that

$$f(x) = \begin{cases} \frac{c}{x^2} & x > 2 \\ 0 & x < 2 \end{cases}$$

is a probability density function.

11. Snow is falling on the ground at the rate of 4 inches/minute. It is melting at a rate of 75% How much snow is on the ground after 5 hours? How much snow remains on the ground if it continues to snow indefinitely?

12. Consider

$$\int_2^\infty \frac{1}{\sqrt{4+x^2}}\,dx$$

   (a) Do you expect this integral to converge or diverge? Explain.

   (b) Calculate the integral.

13. The temperature of a cup of coffee, in accordance with Newton's Law of Heating and cooling, satisfies the differential equation

$$\frac{dH}{dt} = -.2(H - 20)$$

where H is measured in centigrade. The coffee is served at 90 degrees. How long must we wait to drink it, if it should be 50 degrees when we do?

14. Suppose that you are on hold, and that your anticipated wait time is modeled by the probability density function $f(x) = \frac{8}{(x+2)^3}$, where x is your wait time in minutes.

    (a) Explain why f is a suitable probability density function.

    (b) What is the probability that your call is answered in the first minute?

15. Define a sequence $\{a_n\}_{n=1}^{\infty}$ by $a_1 = 1$ and $a_n = a_{n-1}^2 - 1$. What are the first ten terms of the sequence? Does the sequence approach a limit? If so, what? If we define $b_n = \frac{a_n}{n!}$, does the series $\sum_{n=1}^{\infty} b_n$ converge? If so, to what?

16. Determine the convergence or divergence of the following series.

    (a)
    $$\sum_{n=1}^{\infty} \frac{n^2 - 1}{2 - n^3}$$

    (b)
    $$\sum_{n=1}^{\infty} \ln\left(\frac{2n}{n-3}\right)$$

    (c)
    $$\sum_{n=1}^{\infty} \frac{n^{2n}}{(1 + 2n^2)^n}$$

    (d)
    $$\sum_{n=1}^{\infty} \frac{(-1)^n n + 1}{2n^2 + 1}$$

    (e)
    $$\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$$

(f)

$$\sum_{n=1}^{\infty} \frac{3^n}{4^n + 5^n}$$

17. Determine the values of x for which

$$\sum_{n=0}^{\infty} \frac{n}{n^2+1}(1-x)^n$$

converges. Be sure to check endpoints.

18. Determine the convergence or divergence of each of the following series

(a)

$$\sum_{n=1}^{\infty} \frac{\ln n}{n^{\frac{3}{2}}}$$

(b)

$$\sum_{n=1}^{\infty} \frac{n!}{10^n}$$

(c)

$$\sum_{n=1}^{\infty} \frac{\cos\left(\frac{1}{n}\right)}{n}$$