

Fall 2012
Math 126: Practice Midterm

This problem set represents the types of problems that you're likely to see on Friday's midterm. The inclusion or omission of material from this set does not guarantee that said material will or will not be on the midterm. In practice, this set is longer than what you should expect on Friday.

1. Approximate the area under $y = t^2 - t^3$ between $x = 0$ and $x = 1$ using 4 subdivisions. Give an over- and under-estimate. Check that your answers make sense by finding the exact area.
2. Find the area under the line $y = 2x + 3$ between $x = 1$ and $x = 4$ using
 - (a) Geometry.
 - (b) Calculus.
3. $\int \frac{x^3+1}{x^2} dx$
4. $\int (x + 3) \cos(x^2 + 6x + 5) dx$
5. $\int_0^a \frac{x dx}{\sqrt{1-x^2}}$. What are the restrictions on a ?
6.
$$\int \frac{2x}{x^2 - 4x + 3} dx$$
7.
$$\int \sec^3(x) \tan^3(x) dx$$
8.
$$\int \frac{x - 1}{\sqrt{1 - x^2}} dx$$
9.
$$\int \frac{\sqrt{x} - 1}{\sqrt{x} + 1} dx$$
10.
$$\int \frac{\ln x}{\sqrt{x}} dx$$
11.
$$\int e^{2x} \cos(2x) dx$$
12. Find the volume of the solid generated when the region bounded by the curve $y = x \ln x$ between $x = 2$ and $x = 5$ is rotated about the y axis.
13. Consider the area under the curve $y = e^x$ between $x = 0$ and $x = \ln 3$
 - (a) Find c such that the line $x = c$ bisects this area. Is c greater than, less than, or equal to $\frac{1}{2} \ln 3$? Explain graphically.

- (b) Compute the volume generated by revolving this area about
- the x axis. (solve this integral...)
 - the y axis. (...but not this one.)

14. Suppose that the acceleration of an object is given by $a(t) = 3t^2 - 2t$, with initial velocity equal to 0 m/s and initial position given by 4 m. Where is the object when it comes to rest for the first time after $t = 0$?

15. Find

$$\int_{\sqrt{3}}^{\infty} \frac{2}{1+x^2}$$

if it exists. Use this result to determine whether

$$\int_{\sqrt{3}}^{\infty} \frac{e^{-x}}{1+x^2}$$

converges or diverges.

16. Determine

$$\int_2^{\infty} \frac{1}{x^2-1} dx$$

converges or diverges.

17. A tank is in the shape of a circular cylinder laying on its side, 4 feet long and 2 feet high (i.e. the cylinder has radius 1). Find the amount of work done in pumping water out of the full tank (Recall that water weighs 62 lb/ft³.)