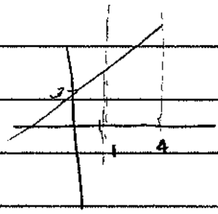
Math 126 : Practice Midterm, Even Problems

2. $y = 2x+3$ $\quad x=1,\ x=4$



Geometry: $A_{trapezoid} = \frac{1}{2}(b_1+b_2)h$

$\frac{1}{2}(11+5)3 = \boxed{24}$

Calculus: $\int_1^4 2x+3\,dx = x^2+3x\Big]_1^4$

$(16+12) - (1+3)$

$28 - 4 = \boxed{24}$

4. $\int (x+3)\cos(x^2+6x+5)\,dx$

$u = x^2+6x+5$

$du = 2x+6$

$\frac{1}{2}du = (x+3)dx$

$\frac{1}{2}\int \cos u\,du$

$\frac{1}{2}\sin u + C$

$\boxed{\frac{1}{2}\sin(x^2+6x+5) + C}$

6. $\int \frac{2x}{x^2-4x+3}dx = \int \frac{2x}{(x-1)(x-3)} = \int \frac{A}{x-3} + \frac{B}{x-1}\,dx$

$A(x-1) + B(x-3) = 2x$

$Ax - A + Bx - 3B = 2x$

$A+B = 2 \qquad -A-3B = 0$

$-A-3B = 0$

$-2B = 2;\ B = -1$

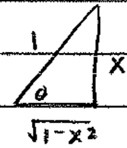
$A = 3$

$\int \frac{3}{x-3} - \frac{1}{x-1}\,dx = \boxed{3\ln|x-3| - \ln|x-1| + C}$

8. $\int \frac{x-1}{\sqrt{1-x^2}}\,dx$ $\quad x = \sin\theta$, $dx = \cos\theta\,d\theta$ $\quad$ OR: $\int \frac{x}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}}\,dx$

↑ usub ↑ trig sub

$\int \frac{(\sin\theta-1)\cos\theta\,d\theta}{\sqrt{1-\sin^2\theta}}$



$\int (\sin\theta - 1)\,d\theta$

$-\cos\theta - \theta + C$

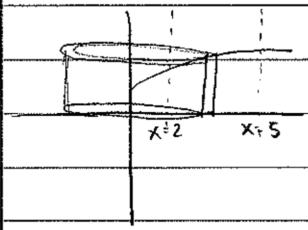
$-\sqrt{1-x^2} - \arcsin x + C$

10. $\int \frac{\ln x}{\sqrt{x}}\,dx = \int x^{-1/2}\ln x\,dx$ $\quad u = \ln x \quad v = 2x^{1/2}$

$= 2x^{1/2}\ln x - \int 2x^{1/2}\left(\frac{1}{x}\right)dx$ $\quad du = \frac{1}{x}dx \quad dv = x^{-1/2}$

$= 2x^{1/2}\ln x - \int 2x^{-1/2}\,dx$

$= \boxed{2x^{1/2}\ln x - 4x^{1/2} + C}$

12. 

cylindrical shells

$V_{shell} = 2\pi x(x\ln x)dx$

$V_{shell} = 2\pi\int_2^5 x^2\ln x\,dx$

$V_{shell} = 2\pi\left[\frac{x^3}{3}\ln x - \frac{x^3}{9}\right]_2^5$

$2\pi\left[\left(\frac{5^3}{3}\ln 5 - \frac{5^3}{9}\right) - \left(\frac{2^3}{3}\ln 2 - \frac{2^3}{9}\right)\right]$

* strip // to axis → shells

strip ⊥ to axis → washers or disks

1. $y = t^2 - t^3$

x	0	$1/4$	$1/2$	$3/4$	1
$f(x)$	0	$\frac{3}{64}$	$\frac{1}{8}$	$\frac{9}{64}$	0

UL $\frac{1}{4}\left(\frac{3}{64}\right) + \frac{1}{4}\left(\frac{1}{8}\right) + \frac{1}{4}\left(\frac{9}{64}\right) = 5/64$

LL $\frac{1}{4}\left(\frac{3}{64}\right) + \frac{1}{4}\left(\frac{1}{8}\right) + \frac{1}{4}\left(\frac{9}{64}\right) = 5/64$

$$\int_0^1 t^2 - t^3 \, dt$$

$$= \frac{1}{3}t^3 - \frac{1}{4}t^4 \Big|_0^1$$

$$= \frac{1}{12}.$$

3. $\displaystyle\int \frac{x^3+1}{x^2} \, dx$

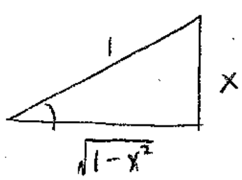
$$= \int \frac{x^3}{x^2} dx + \int \frac{1}{x^2} \, dx$$

$$= \int x + x^{-2} \, dx$$

$$= \frac{x^2}{2} + \frac{x^{-1}}{-1} + C$$

$$= \frac{x^2}{2} - \frac{1}{x} + C$$

5. $\displaystyle\int_0^a \frac{x\,dx}{\sqrt{1-x^2}}$



let $x = \sin\theta$

$dx = \cos\theta \, d\theta$

$$\int \frac{\sin\theta}{\sqrt{1-\sin^2\theta}} \cos\theta \, d\theta$$

$$= \int \sin\theta \, d\theta$$

$$= -\cos\theta \, d\theta$$

$$= -\sqrt{1-x^2} \Big|_0^a$$

$$= -\sqrt{1-a^2} + 1$$

$$\boxed{-1 \le a \le 1}$$

Practice Midterm (Olin 157).

7. $\displaystyle\int \sec^3 x \cdot \tan^3 x \, dx$

$$= \int \sec^2 x \cdot \tan^2 x \quad \sec x \tan x \, dx$$

let $u = \sec x$

$du = \sec x \tan x \, dx$

$$= \int u^2 \cdot (u^2 - 1) \, du$$

$$= \int u^4 - u^2 \, du$$

$$= \frac{1}{5}u^5 - \frac{1}{3}u^3 + C$$

$$= \frac{1}{5}(\sec^5 x) - \frac{1}{3}(\sec^3 x) + C$$

$$\int_0^a \frac{x\,dx}{\sqrt{1-x^2}}$$

let $u = 1 - x^2$

$du = -2x$

$$= \int_0^a -\frac{1}{2} \cdot \frac{1}{\sqrt{u}} \, du$$

$$= -\frac{1}{2} \int_0^a u^{-1/2} \, du$$

$$= -\frac{1}{2}(2) u^{1/2} \Big|_0^a$$

$$= -u^{1/2} \Big|_0^a$$

$$= -\sqrt{1-x^2} \Big|_0^a$$

9. $\int \frac{\sqrt{x}-1}{\sqrt{x}+1}\,dx$

let $y=\sqrt{x}$; $x=y^2$

$dx=2y\,dy$

$=\int \frac{y-1}{y+1}(2y)\,dy$

$=\int \frac{2y^2-2y}{y+1}\,dy$

$$\begin{array}{r|l} & 2y-4 \\ \hline y+1 & 2y^2-2y \\ & 2y^2+2y \\ \hline & -4y \\ & -4y-4 \\ \hline & 4 \end{array}$$

$=\int 2y-4+\frac{4}{y+1}\,dy$

$=y^2-4y+4\ln|y+1|+C$

$=x-4\sqrt{x}+4\ln|\sqrt{x}+1|+C$

11. $\int e^{2x}\cos(2x)\,dx$

let $u=e^{2x}$ $\quad dv=\cos 2x$

$du=2e^{2x}\,dx$ $\quad v=\frac{1}{2}\sin 2x\,dx$

$\int e^{2x}\cos(2x)\,dx = e^{2x}\frac{1}{2}\sin 2x - \int e^{2x}\sin 2x\,dx$

let $u=e^{2x}$ $\quad dv=\sin 2x\,dx$

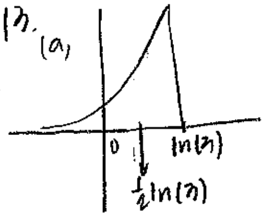
$du=2e^{2x}$ $\quad v=-\frac{1}{2}\cos 2x$

$\int e^{2x}\cos(2x)\,dx = e^{2x}\frac{1}{2}\sin 2x+(e^{2x}\cos 2x)(\frac{1}{2})-\int \frac{1}{2}\cos 2x\,(2e^{2x})$

$2\int e^{2x}\cos 2x = \frac{1}{2}e^{2x}\sin 2x+\frac{1}{2}e^{2x}\cos 2x$

$\int e^{2x}\cos 2x = \frac{1}{4}\left(e^{2x}\sin 2x+e^{2x}\cos 2x\right)+C$

13. (a)



Greater than $\frac{1}{2}\ln(3)$

Total $=\int_0^{\ln 3} e^x\,dx = e^x\Big|_0^{\ln 3} = 2$

$\frac{1}{2}$ area $=1=\int_0^c e^x\,dx$

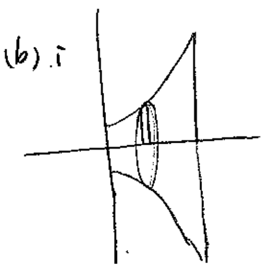
$e^x\Big|_0^c=1$

$e^c-e^0=1$

$e^c=2$

$c=\ln(2)$

(b) i



$V_{Disk}=\pi r^2\cdot h$

$=\pi(y^2)\,dx$

$=\pi\int_0^{\ln 3}(e^x)^2\,dx$

$=\pi\left(\frac{e^{2x}}{2}\right)\Big|_0^{\ln 3}$

$=\pi\left(\frac{e^{2\ln 3}}{2}-\frac{e^0}{2}\right)$

$=\frac{9\pi}{2}-\frac{\pi}{2}$

$=4\pi$

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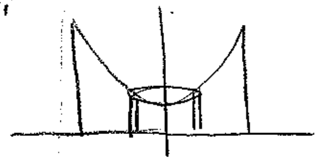

$\int_0^{\ln 3} 2\pi x e^x \, dx$

14) $a(t) = 3t^2 - 2t$; $v(t) = \int 3t^2 - 2t \, dt$

$\Rightarrow v(t) = t^3 - t^2 + v_0 = t^3 - t^2 + 0$

$s(t) = \int t^3 - t^2 dt = \frac{t^4}{4} - \frac{t^3}{3} + s_0$

$= \frac{t^4}{4} - \frac{t^3}{3} + 4$

object is at rest when $v = 0$

$t^3 - t^2 = 0 \Rightarrow t = 1$

$s(1) = \frac{1}{4} - \frac{1}{3} + 4 = 3\frac{11}{12}$.

15. $\int_{\sqrt{3}}^{\infty} \frac{2}{1+x^2} = \lim_{b \to \infty} \int_{\sqrt{3}}^{b} \frac{2}{1+x^2} dx = \lim_{b \to \infty} 2 \arctan x \Big]_{\sqrt{3}}^{b}$

$= \lim_{b \to \infty} 2 \arctan b - 2 \arctan \sqrt{3} = 2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{3}\right) = \frac{\pi}{3}$.

Since $\frac{e^{-x}}{1+x^2} < \frac{2}{1+x^2}$ on $(\sqrt{3}, \infty)$, we have

$\int_{\sqrt{3}}^{\infty} \frac{e^{-x}}{1+x^2} dx < \int_{\sqrt{3}}^{\infty} \frac{2}{1+x^2} dx$

So $\int_{\sqrt{3}}^{\infty} \frac{e^{-x}}{1+x^2} dx$ converges

16 $\int_2^{\infty} \frac{1}{x^2-1} dx = \lim_{b \to \infty} \int_2^b \frac{1}{x^2-1} dx = \lim_{b \to \infty} \int_2^b \frac{-\frac{1}{2}}{x+1} + \frac{\frac{1}{2}}{x-1} dx$

$\frac{A}{x+1} + \frac{B}{x-1}$

$Ax - A + Bx + B = 1$

$A + B = 0$

$B - A = 1$

$B = 1/2$, $A = -1/2$

$= \lim_{b \to \infty} \frac{1}{2} \ln \left| \frac{x-1}{x+1} \right| \Big]_2^b$

$= \frac{1}{2} (\ln 1) - \ln\left(\frac{1}{3}\right)$

$= -\frac{1}{2} \ln \frac{1}{3} = \frac{1}{2} \ln 3$.

$= \ln \sqrt{3}$.

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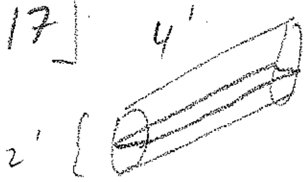
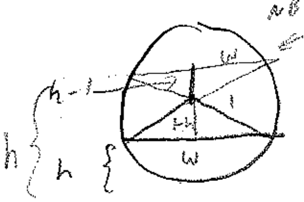
Break the tank into horizontal slabs...



$$V_{slab} = l \cdot w \cdot h$$

NB $(h-1)^2 = (1-h)^2$

$$= 4(2\sqrt{1-(1-h)^2})\,dh$$



$$F_{slab} = 62 \cdot 4 \cdot 2\sqrt{1-(1-h)^2}\,dh$$

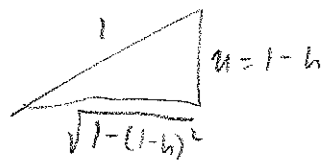
$$W_{slab} = 62 \cdot 4 \cdot 2\sqrt{1-(1-h)^2} \cdot (2-h)\,dh$$

$$W_{TOTAL} = 496\int_0^2 (2-h)\sqrt{1-(1-h)^2}\,dh$$

let $u = 1-h$

$du = -dh$

$$-496\int_0^2 (1+u)\sqrt{1-u^2}\,du$$



$u = \sin\theta$

$du = \cos\theta\,d\theta$

$$-496\int_{h=0}^{h=2} (1+\sin\theta)\sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$$-496\int_{h=0}^{h=2} \cos^2\theta + \sin\theta\cos^2\theta\,d\theta$$

$$= -496\int_{h=0}^{h=2} \frac{1}{2} + \frac{\cos 2\theta}{2} + \sin\theta\cos^2\theta\,d\theta$$

$$= -496\left[\frac{\theta}{2} + \frac{\sin 2\theta}{4} + \frac{\cos^3\theta}{3}\right]_{h=0}^{h=2}$$

$$-496\left[\frac{\arcsin(1-h)}{2} + \frac{(1-h)\sqrt{1-(1-h)^2}}{2} + \frac{\left(\sqrt{1-(1-h)^2}\right)^3}{3}\right]_0^2$$

$$= -496\left[\arcsin(-1) - \arcsin(1)\right] = \underline{496\pi}$$