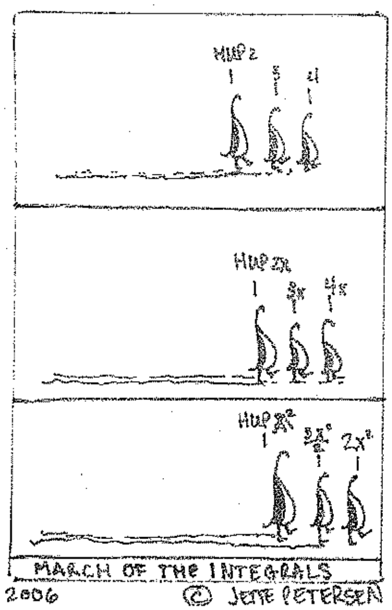
KEY

Math 126: Quiz 1

September 7, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Find the area under $f(x) = 3x$ from $x = 2$ to $x = 4$ using only high-school geometry.

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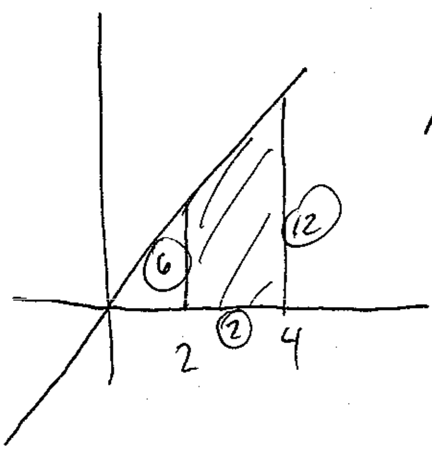


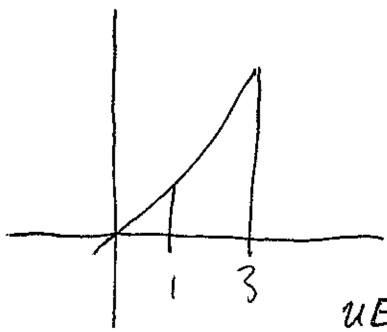
Area = Trapezoid

$$= \frac{1}{2} h (b_1 + b_2)$$

$$= \frac{1}{2}(2)(6+12) = 18.$$

2. Let $f(x) = x^2 + x$.

(a) Give over- and under-estimates of the area under $f(x)$ between $x = 1$ and $x = 3$, using 4 subdivisions. $(1,3)$ $n=4 \to \Delta x = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$



x	1	1.5	2	2.5	3
$f(x)$	2	$\frac{15}{4}$	6	$\frac{35}{4}$	12

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UE: $\frac{1}{2}\left(2 + \frac{15}{4} + 6 + \frac{35}{4}\right) = \frac{1}{2}\left(8 + \frac{50}{4}\right)$

$= \frac{82}{8} = \frac{41}{4}$ (10.25)

OE: $\frac{1}{2}\left(\frac{15}{4} + 6 + \frac{35}{4} + 12\right) = \frac{1}{2}\left(18 + \frac{50}{4}\right) = \frac{122}{8} = \frac{61}{4}$ (15.25)

(b) Find this exact area by solving an appropriate definite integral.

$$\int_1^3 x^2 + x \, dx = \frac{x^3}{3} + \frac{x^2}{2} \Bigg|_1^3$$

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$$= \frac{27}{3} + \frac{9}{2} - \frac{1}{3} - \frac{1}{2}$$

$$= \frac{26}{3} + 4 = \frac{38}{3} \quad (12.\overline{666})$$

3. Use the Fundamental Theorem of Calculus to find

$$\frac{d}{dx}\int_2^{3x^2} \frac{1}{\sqrt{t^2+1}}\,dt.$$

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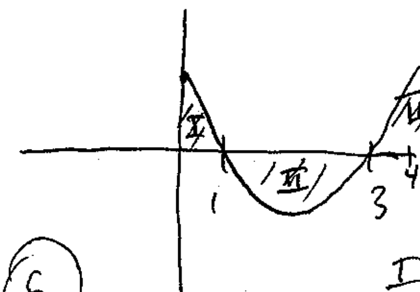
$$\frac{d}{dx}\int_2^{3x^2}\frac{1}{\sqrt{t^2+1}}\,dt = \frac{d}{dx}\left[F(3x^2)-F(2)\right] \qquad \text{Where } F'(t)=\frac{1}{\sqrt{t^2+1}}$$

$$= 6x\, F'(3x^2) - 0$$

$$= 6x \cdot \frac{1}{\sqrt{9x^4+1}}$$

4. Suppose that a particle moves with velocity given by $v(t) = t^2 - 4t + 3$ m/s. Find both the *total* distance travelled, and the *net* distance travelled over the first 4 seconds. (Note: You will find it helpful to plot $v(t)$.)



$$v(t) = t^2 - 4t + 3 = (t-1)(t-3)$$

NET: $\int_0^4 t^2 - 4t + 3\,dt = \frac{t^3}{3} - 2t^2 + 3t \Big|_0^4$

$$= \frac{64}{3} - 32 + 12 = \frac{64}{3} - \frac{60}{3} = \frac{4}{3} \text{ m}$$

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$$\text{I} \quad - \quad \text{II} \quad + \quad \text{III}$$

TOTAL: $\int_0^1 v(t)\,dt - \int_1^3 v(t)\,dt + \int_3^4 v(t)\,dt = \frac{t^3}{3} - 2t^2 + 3t \Big|_0^1 = \frac{4}{3}$

$$\Big|_1^3 = 9 - 18 + 9 - \frac{4}{3} = -\frac{4}{3}$$

$$\Big|_3^4 = \frac{4}{3} - 0 = \frac{4}{3}$$

$$= \frac{4}{3} + \frac{4}{3} + \frac{4}{3} = 4 \text{ m}$$

5. Bonus: Give two examples of functions such that $f'(x) = f(x)$. Half a point for the first, up to 1.5 points for the second.

$f(x) = f'(x) = e^x$ (easy!)

$f(x) = f'(x) = 6e^x$ or πe^x or any Ce^x.

Note! This is a "shift" from our usual mindset of the $+C$.