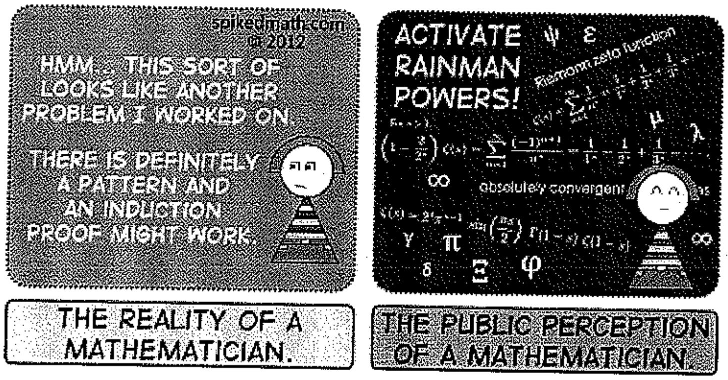
KEY

Math 126: Quiz 3

October 5, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!



1. Find

$$\int \frac{dx}{x^2 + 2x - 15}$$

$$\int \frac{dx}{x^2+2x-15} = \int \frac{dx}{(x+5)(x-3)} = \int \frac{A}{x+5} + \frac{B}{x-3}\, dx$$

$$A(x-3) + B(x+5) = 1$$

$$(A+B)x + (-3A+5B) = 1$$

$$A+B=0$$

$$\underline{-3A+5B=1}$$

$$8B=1, \quad B = 1/8 \quad A = -1/8$$

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$$\int \frac{dx}{x^2+2x-15} = \int \frac{-\frac{1}{8}}{x+5} + \frac{\frac{1}{8}}{x-3}\, dx = -\frac{1}{8}\ln|x+5| + \frac{1}{8}\ln|x-3| + C$$

2. A particle moving in a straight line has velocity $v(t) = t^3 - t^2$ meters/sec. (Please give proper units for your answers)

(a) If the position at 0 seconds is 2 meters, find the position after 2 seconds.

$v(t) = t^3 - t^2$

$s(t) = \frac{t^4}{4} - \frac{t^3}{3} + C$ $\quad$ $s(0) = 2 \Rightarrow C = 2$

$s(t) = \frac{t^4}{4} - \frac{t^3}{3} + 2$; $s(2) = \frac{2^4}{4} - \frac{2^3}{3} + 2 = 4 - \frac{8}{3} + 2$

$= 10/3$ meters

(b) Find the acceleration after 2 seconds.

$\frac{d}{dt}$

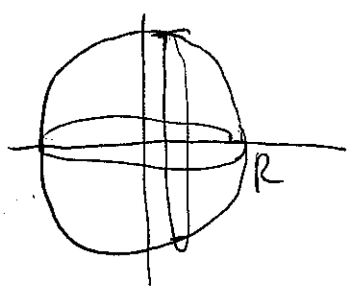
5

$v(t) = t^3 - t^2$

$a(t) = 3t^2 - 2t$

$a(2) = 12 - 4 = 8\ m/s^2$

3. Find the volume of a sphere of radius R by rotating the circle $x^2 + y^2 = R^2$ about either coordinate axis. You may use any method that you wish, but be clear about your methods.



Using Disks

$V_{DISK} = \pi r^2 h = \pi y^2 dx$

$= \pi (R^2 - x^2) dx$

$V_{total} = \int_{-R}^{R} \pi\ R^2 - x^2\ dx$

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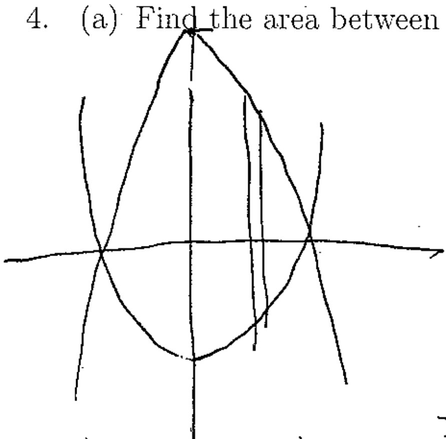
$= \pi \left[R^2 x - \frac{x^3}{3} \right]_{-R}^{R}$

$= \pi \left(\left(R^3 - \frac{R^3}{3}\right) - \left(-R^3 - \frac{R^3}{3}\right) \right)$

$= \pi \left(\frac{2R^3}{3} + \frac{2R^3}{3} \right) = \frac{4}{3} \pi R^3$

4. (a) Find the area between the curves $f(x) = x^2 - 4$ and $g(x) = 8 - 2x^2$.



$$A = \int_a^b g(x) - f(x)\,dx$$

$$= \int_{-2}^{2} (8-2x^2)-(x^2-4)\,dx$$

$$= \int_{-2}^{2} 12 - 3x^2\,dx = 12x - x^3 \Big|_{-2}^{2}$$

$$= (24-8)-(-24-(-8))$$

$$= 32$$

$(x^2-4) = (8-2x^2)$

$\to 3x^2 = 12$

$x^2 = 4$; $x = \pm 2$

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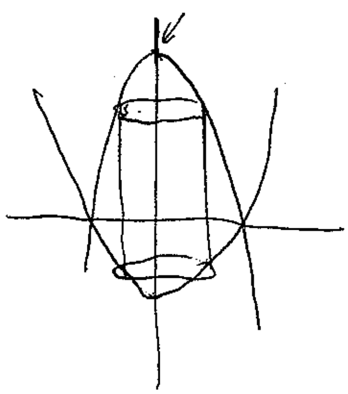
(b) Find the volume generated when this area is revolved around the y-axis.

$V_{shell} = 2\pi r h\,dr = 2\pi x\,(g(x)-f(x))\,dx$

Shells!



$$2\pi \int_0^2 x\,(8-2x^2-(x^2-4))\,dx$$

$$= 2\pi \int_0^2 x(12-3x^2)\,dx$$

$$= 2\pi \int_0^2 12x - 3x^3\,dx$$

$$= 2\pi \left(6x^2 - \tfrac{3}{4}x^4\right]_0^2$$

$$= 2\pi(24-12) = 24\pi$$

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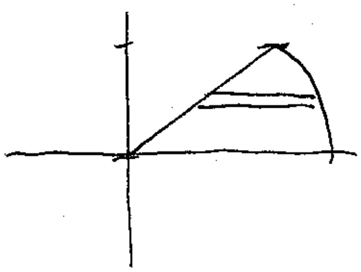
For 5 and 6: Your integral should include the appropriate bounds and be in terms of one variable.

5. Set up, but DO NOT COMPUTE the integral for the area bound by $y = 0$, $x = y$ and $x^2 + y^2 = 1$.



$$\int_0^{\frac{\sqrt{2}}{2}} \sqrt{1-y^2} - y \; dy$$

$x^2 + y^2 = 1$

$x = 1 \rightarrow$

$2x^2 = 1 \quad x^2 = \frac{1}{2} \quad x = \frac{\sqrt{2}}{2} \quad y = \frac{\sqrt{2}}{2}$

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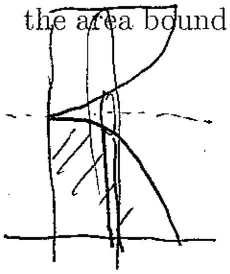
6. Set up, but DO NOT COMPUTE the integral for the volume generated when the area bound by $y = \cos(x)$, $y = 0$ and $x = 0$ is rotated about the line $y = 1$.



Washers!

$$V_{washer} = \pi (R_{out}^2 - R_{in}^2)\, dh$$

$$= \pi \; 1^2 - \cancel{(\cos x)^2}\;(1-\cos x)^2 \; dx$$

$$V_{total} = \pi \int_0^{\pi/2} 1^2 - \cos^2 x \; dx$$

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7. (Bonus) Extra Credit:

   (a) When is your birthday? (Month and Day only. No years. Please.) Nov. 19

   (b) Of the 46 of us (in both sections), the approximate probability that two of us have the same birthday is : 12%, 24%, 48%, 95%.