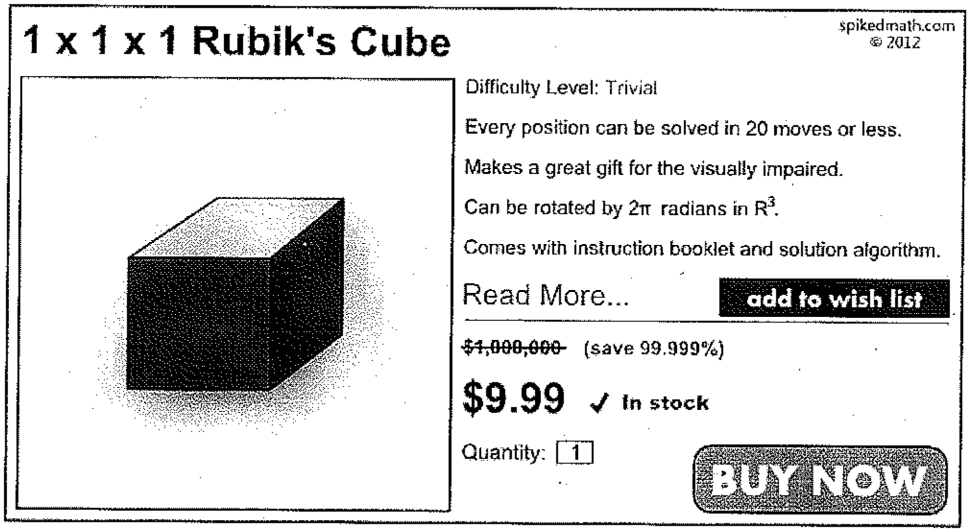
KEY

# Math 126: Quiz 4

November 2, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!




1. Solve the initial value problem $\frac{dy}{dx} = \frac{x}{y}$ subject to the condition $y(1) = 2$.

$$\frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$y^2 = x^2 + C'$$

$$y = \sqrt{x^2 + C'}$$

$$y = \sqrt{x^2+3}$$

$y(1)=2$

$\Rightarrow C' = 3$

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2. Professor $B^2$ calls the cable company to complain that he can't watch football. The company claims that the average wait time on hold is 5 minutes, giving a probability distribution function of $p(x) = 0.2e^{-0.2x}$.

   (a) Find the probability that Prof $B^2$ gets his call answered between 2 and 4 minutes.

$$\int_2^4 0.2e^{-0.2t}\,dt = -e^{-0.2t}\Big|_2^4$$

$$= -e^{-0.8} + e^{-.4}$$

$$p = .221$$

   (b) Find the probability that Prof $B^2$ has to wait more than 15 minutes.

$$\int_{15}^{\infty} 0.2\, e^{-0.2t}\,dt = -e^{-0.2t}\Big|_{15}^{\infty}$$

$$= \lim_{b\to\infty} -e^{-0.2b} + e^{-0.2(15)} = e^{-3} = .0497$$

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   (c) Do most people have their calls answered before 5 minutes? Explain.

Most people have their answered before 5 minutes.
If 5 minutes is the average, it accounts for people who have to wait more than 10, 15, 20 minutes

3. Find the length of the curve $f(x) = 2x^{3/2}$ from $(1, 1)$ to $(4, 16)$.

$$\text{Length}: \int_1^4 \sqrt{1 + f'(x)^2}\, dx = \int_1^4 \sqrt{1 + (3x^{1/2})^2}\, dx$$

$$= \int_1^4 \sqrt{1 + 9x}\, dx = \frac{1}{9}(1+9x)^{3/2} \Big|_1^4$$

$u = 1 + 9x$

$du = 9dx$

$$= \frac{1}{9}\left[\left(37^{3/2} - 10^{3/2}\right)\right]$$

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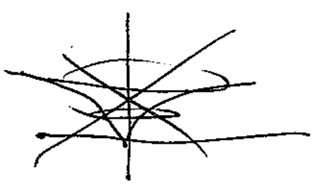
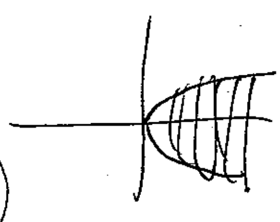
3 Set up, but don't evaluate, the integral to compute the surface area generated when the curve in problem 2 is rotated about the $x$-axis.



$$SA = \int_a^b 2\pi r\, ds$$



$$= \int_1^4 2\pi y \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

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$$= \int_1^4 2\pi \left(2x^{3/2}\right) \sqrt{1 + 9x}\, dx.$$

$$\text{or } \int_1^{16} 2\pi y \sqrt{1 + \left(\frac{1}{3}\left(\frac{y}{2}\right)^{-1/3}\right)^2}\, dy$$

5. Hungary is one of the few countries in the world with a world with a decreasing population. Every year, 2 percent of the current population either dies off or leaves, while only 150,000 new people are born or move there. Hungary's current population is 10,000,000. Set up a differential equation to determine the population as a function of time. When will the population dip below 9,000,000? What is the projected long term population?

$$\frac{dP}{dt} = -0.02P + 150{,}000$$

or, in thousands

$$\frac{dP}{dt} = -0.02P + 150$$

$$\frac{dP}{dt} = -0.02(P - 7500)$$

$$\frac{dP}{P-7500} = -0.02\,dt$$

$$\ln|P-7500| = -0.02t + C$$

$$P = 7500 + Be^{-0.02t}$$

$$B = 2500 \leftarrow 10000 = 7500 + Be^{0}$$

So $P(t) = 7500 + 2500e^{-0.02t}$

$$9000 = 7500 + 2500e^{-0.02t}$$

$$\frac{\ln\frac{1500}{2500}}{-0.02} = \underline{25.54 \text{ years from now}}$$

long term pop → 7.5 million

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6. (Bonus) Find at least two functions $y$ which satisfy the *second order* differential equation $\frac{d^2x}{dy^2} = -y$. (This denotes the *second* derivative.)

$$\frac{d^2y}{dx^2} = -y$$

Think Trig functions!

$y = \cos x$

$y = \sin x$

both work.