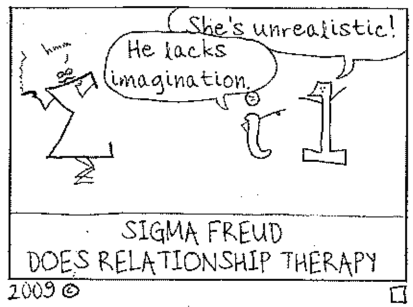
KEY

Math 126: Quiz 5
November 16, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!



1. (a) What is the difference between a *series* and a *sequence.*

A ~~series~~ sequence is an ordered list of terms, while a series is an infinite sum of terms.

(b) True or false: If $\{a_n\}$ converges, then $\sum_{n=1}^{\infty} a_n$ also converges.

false. The sequence may converge, even to 0, while the series diverges

4.5

(c) True or false: If $\sum_{n=1}^{\infty} a_n$ converges, then $\{a_n\}$ also converges.

True. If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$

2. Determine if the following sequences converge. If so, find the limit as n goes to infinity.

(a) $a_n = \frac{4^n}{5^{n+1}}$ $\quad \frac{4}{25}, \frac{16}{125}, \frac{64}{625}, \ldots$

geometric sequence with $r<1 \Rightarrow$ $a_n \to 0$.

(b) $b_n = \frac{2n+1}{\sqrt{5+3n^2}}$ effective powers of n on top & on bottom are equal, (1) so compare coeffs

$b_n \to \frac{2}{\sqrt{3}}$.

4.5

(c) $c_n = \ln(n+1) - \ln(n)$.

$c_n = \ln(n+1) - \ln(n) = \ln\left(\frac{n+1}{n}\right) \to \ln 1$ as $n \to \infty$ so $c_n \to 0$.

3. Write

$$\frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \frac{32}{625} + \ldots$$

in a closed form, and find its value.

This series is geometric, with ratio $\frac{2}{5}$, first term $\frac{4}{5}$

so $Sm = \sum_{n=0}^{\infty} \frac{4}{5}\left(\frac{2}{5}\right)^n$.

$Sum = \frac{a}{1-r} = \frac{4/5}{1-2/5} = 4/3$.

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4. Determine whether each series below converges or diverges. Make sure to explain which tests you're using.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ (a) is alternating, so we look at

Converges (conditionally!) $\lim_{n\to\infty} \frac{1}{\sqrt[3]{n}}$, which is decreasing and going to 0,

thus $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}}$ converges by Alt series test

(b) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ → Divergent →

Integral test: $\int_2^{\infty} \frac{1}{x\sqrt{\ln(x)}}\,dx = \lim_{b\to\infty} \int_{x=2}^{x=b} \frac{du}{\sqrt{u}} = \lim_{b\to\infty} 2u^{1/2}\Big]_2^{\infty}$

let $u = \ln x \to du = \frac{1}{x}dx$

$= \lim_{b\to\infty} 2\sqrt{\ln b} - 2\sqrt{\ln 2}$

goes to ∞.

(c) $\sum_{n=1}^{\infty} \frac{n+5}{2n^3}$ Converge

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Limit comparison test w/ $\sum \frac{1}{n^2}$

$\lim_{n\to\infty} \left(\frac{\frac{n+5}{2n^3}}{1/n^2} \right) = \lim_{n\to\infty} \frac{n^3+5n^2}{2n^3} = \frac{1}{2}$, so both $\sum_{n=1}^{\infty} \frac{n+5}{2n^3}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$

behave the same.

Since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, so does $\sum_{n=1}^{\infty} \frac{n+5}{2n^3}$

(d) $\sum_{n=1}^{\infty} \frac{2^{2n}+1}{3^n}$

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$\sum \frac{2^{2n}}{3^n} + \frac{1}{3^n} = \sum \left(\frac{4}{3}\right)^n + \left(\frac{1}{3}\right)^n$ → Divergent.

sum of geometrics, divergent, since $r > 1$

5. Alice and Bob are playing the following game: They will take turns tossing a fair coin. First 'heads' wins. Alice goes first.

   (a) Who, if anyone, has the advantage in this game? Alice, since she goes first.

   (b) What is the probability that Alice wins on her first turn? $\frac{1}{2}$

   (c) What is the probability that Alice wins on her second turn? (It will help you to think of what needs to happen on the first two tosses of the coin.)

   (TTH) $p = 1/8$

   (d) What is the probability that Alice wins on her third turn?

   TTTTH $p = 1/32$

   (e) Express Alice's overall probability of winning as a geometric series, and find its sum.

   $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots$ geometric $a = \frac{1}{2}$ $r = \frac{1}{4}$

   $$\frac{\frac{1}{2}}{1-\frac{1}{4}} = \frac{\frac{1}{2}}{3/4} = 2/3.$$

   (f) Find Bob's probability of winning both by calculating an appropriate geometric series, and by subtracting Alice's probability from the total probability.

   Bobs prob $= 1 - 2/3 = 1/3$

   Bobs Prob $= 1/4 + 1/16 + 1/64 + \dots = \frac{1/4}{1-1/4} = \underline{1/3}$

   TH TTTH TTTTTH

(4)

Bonus: Explain the following joke.

What did the mathematician say after Thanksgiving?

$$\sqrt{\frac{-1}{64}} = \frac{i}{8}$$

"I over eight". :)

(1)