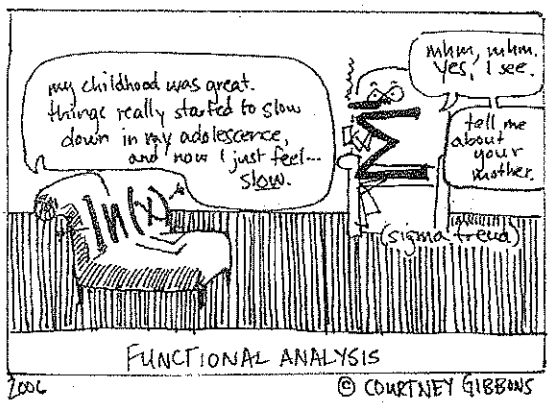
KEY

Math 126: Quiz 6
December 7, 2012

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!



1. Determine the whether each series converges or diverges.

(a) $\sum_{n=0}^{\infty} \frac{3^n}{(n+1)!}$

Ratio test $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n\to\infty} \left| \frac{3^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{3^n} \right|$

$= \lim_{n\to\infty} \left| \frac{3}{n+2} \right| = 0$

so the series is Abs. Convergent

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(b) $\sum_{n=1}^{\infty} \left(\frac{n+1}{n^2} \right)^n$

Root test $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{n+1}{n^2}\right)^n} = \lim_{n\to\infty} \frac{n+1}{n^2} = 0$

So the series is Absolutely Convergent

2. Find the interval and radius of convergence of the following series. Remember to check the endpoints!

$$\sum_{n=0}^{\infty} \frac{2n(x-3)^n}{4^n}$$

Ratio Test $\lim_{n\to\infty} \left| \frac{2(n+1)(x-3)^{n+1}}{4^{n+1}} \cdot \frac{4^n}{2n(x-3)^n} \right|$

$= \lim_{n\to\infty} \left| \frac{n+1}{n} \cdot \frac{x-3}{4} \right| = \left| \frac{x-3}{4} \right|$

$\left|\frac{x-3}{4}\right| < 1 \Rightarrow -1 < \frac{x-3}{4} < 1 \Rightarrow -1 < x < 7$

So our interval is $(-1, 7)$

If $x = -1$, $\sum_{n=0}^{\infty} \frac{2n(-4)^n}{4^n}$ is Divergent

If $x = 7$, $\sum_{n=0}^{\infty} \frac{2n(7-3)^n}{4^n} = \sum_{n=0}^{\infty} 2n \Rightarrow$ Divergent

So our interval is

$(-1, 7)$

(4)

So our interval is

? $-1, 7$?

3. Find the series centered at $x = 0$ (write the first few terms and the closed form) for $f(x) = \cos(2x)$ using Taylor's formula. Check your answer against the series for $\cos(x)$ from your toolbox.

$\cos 2x = 1 - \frac{4x^2}{2!} + \frac{16x^4}{4!} - \frac{64x^6}{6!} + \cdots$

$f^n(x)$	$f^n(0)$	c_n
$\cos 2x$	1	$1/0!$
$-2\sin 2x$	0	0
$-4\cos 2x$	-4	$-4/2!$
$8\sin 2x$	0	0
$16\cos 2x$	16	$16/4!$
$-32\sin 2x$	0	0
	-64	$-64/6!$
	⋮	⋮

$\sum_{n=0}^{\infty} (-1)^n \frac{2^n x^{2n}}{2n!}$

$\sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}$

(3)

4. (a) List the first 5 terms of the series for $f(x) = e^x$ (You may pull this one directly from the toolbox) @ $x=0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

(b) List the first 5 non-zero terms from the series for $f(x) = e^{-x^2}$

$$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots$$

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(c) Determine a series for $\int e^{-x^2}\,dx$

$$\int e^{-x^2}\,dx = \int 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \cdots\,dx$$

$$= \left[C + x - \frac{x^3}{3} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} - \cdots \right]$$

$$= C + \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)\,n!}$$

5. What is the value of the sum

$$S = \frac{1}{2} - \frac{1}{6} + \frac{1}{10} - \frac{1}{14} + \ldots ?$$

(Give an exact mathematical answer here, not merely a decimal approximation).

$\arctan x = x - \frac{x}{3} + \frac{x}{5} - \frac{x}{7} + \cdots$

$S = \frac{1}{2} - \frac{\frac{1}{2}}{3} + \frac{\frac{1}{2}}{5} - \frac{\frac{1}{2}}{7} + \cdots = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right)$

$= \frac{1}{2}\arctan(1)$

$= \pi/8$

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6. Using Calculus and a known geometric series, find a series for $f(x) = -\ln(3 - x)$ centered at $x = 0$.

$f(x) = -\ln(3-x)$

$f'(x) = -\frac{1}{3-x}\cdot(-1) = \frac{1}{3-x} = \frac{1}{3(1-x/3)} = \frac{1/3}{1-x/3}$

$= \frac{1}{3} + \frac{x}{9} + \frac{x^2}{27} + \frac{x^3}{81} + \frac{x^4}{243} + \cdots$

So $-\ln(3-x) = \int \frac{+1}{3-x}\,dx = C + \frac{x}{3} + \frac{x^2}{18} + \frac{x^3}{81} + \frac{x^4}{324} + \frac{x^5}{1215} + \cdots$

$-\ln 3$

$C = -\ln(3-0) = -\ln 3$

(4)

7. (Bonus) Which topic are you most confident about heading into the final? Which are you least confident about?

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