Math 126: Practice Midterm

These questions are taken (largely) from past exams. They are meant to represent the types of questions that you will see on the midterm exam, but are not meant to be a comprehensive list. Look to past quizzes, past homework assignments, and class notes for other study resources in addition to these exercises.

1. Tell whether each series converges conditionally, converges absolutely, or diverges. Be sure to use an appropriate test to justify your answers.

   (a)
   $$\sum_{n=0}^{\infty} \frac{\ln n}{2n+1}$$

   (b)
   $$\sum_{n=0}^{\infty} \frac{2n+1}{\sqrt{3n^2+2}}$$

   (c)
   $$\sum_{n=0}^{\infty} \left(\frac{2n+1}{4n-2}\right)^n$$

   (d)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+1}}$$

   (e)
   $$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^3}$$

   (f)
   $$\sum_{n=1}^{\infty} \frac{\sin(n)}{\sqrt{n^3+1}}$$

2. Is $f(x) = \sin(x^2)$ an even or an odd function? What about $f(x) = \sin(x^3)$. Use Taylor Series to justify your answers.

3. Calculate the following sums.

   (a) $4 + \frac{4}{3} + \frac{4}{9} + \ldots$

   (b) $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \ldots$

   (c) $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \ldots$

4. For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n+1}}$$

   converge? Be sure to check endpoints.

5. Determine a series for $f(x) = e^{x^2}$ and use your series to determine

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2}}{x^6}$$

6. Find

$$\frac{d}{dx} \int_{2x}^{3x} \frac{1}{\sqrt{1+t^2}}\, dt$$

7. Find the following integrals and antiderivatives:

   (a)

$$\int_{-2}^{2} (x+1)(x^2+1)\, dx$$

   (b)

$$\int \cos^2(3x)\, dx$$

   (c)

$$\int \frac{1}{1+x}\, dx$$

   (d)

$$\int \frac{\ln(2x+4)}{x+2}\, dx$$