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a) $\sum_{n=1}^{\infty} \frac{2n(n)}{2n+1} > \sum_{n=1}^{\infty} \frac{1}{2n+1} \sim \sum_{n=1}^{\infty} \frac{1}{n}$ Diverges

↑ comparison test ↑ limit comparison test

If $a_n \geq b_n$ and b_n diverges, a_n also diverges

$\lim_{n \rightarrow \infty} \left(\frac{1}{\frac{1}{2}} \right) = \frac{1}{2}$ Finite non-zero so series behave the same

b) $\sum_{n=0}^{\infty} \frac{2n+1}{\sqrt{3n^2+2}}$ $\lim_{n \rightarrow \infty} \frac{2n+1}{\sqrt{3n^2+2}} = \frac{2}{\sqrt{3}} \neq 0$ so series diverges ($\lim_{n \rightarrow \infty} a_n \neq 0$)

c) $\sum_{n=0}^{\infty} \left(\frac{2n+1}{4n-2} \right)^n$

$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = L$ $L > 1$ D
 $L < 1$ C
 $L = 1$?

$\lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{2n+1}{4n-2} \right|^n} = \lim_{n \rightarrow \infty} \frac{2n+1}{4n-2} = \frac{1}{2}$

converges absolutely

d) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{n^2+1}}$ Alternating series test

$\lim_{n \rightarrow \infty} \frac{1}{(n^2+1)^{1/3}} = 0$ so series converges somehow

Rewrite series w/o alternator:

$\sum_{n=1}^{\infty} \frac{1}{(n^2+1)^{1/3}}$ effective power is $n^{2/3}$ where $p < 1$ so diverges

$\sim \sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

Since series diverges w/o alternator, this is conditionally convergent

e) $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^3}$ Ratio test $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = L$ $L < 1$ AC $L = 1$?
 $L > 1$ D

$\lim_{n \rightarrow \infty} \left| \frac{(2(n+1))!}{((n+1)!)^3} \cdot \frac{(n!)^3}{(2n)!} \right| = \lim_{n \rightarrow \infty} \frac{(2n+2)! \cdot n! \cdot n! \cdot n!}{(2n)! \cdot (n+1)! \cdot (n+1)! \cdot (n+1)!} = \lim_{n \rightarrow \infty} \frac{(2n+1)(2n+2)}{(n+1)^3} = 0$

Series is absolutely convergent

Remember that last step is considering limit of sequence, not series

f) $\sum_{n=1}^{\infty} \frac{\sin(n)}{\sqrt{n^3+1}}$ $\leq \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}} \sim \frac{1}{n^{3/2}}$ which converges because $p > 1$
 (because $|\sin(n)| \leq 1$)

absolutely convergent

because abs value of series would also converge since $|\sin(n)| \leq 1$

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2. $f(x) = \sin(x^2)$ $f(x) = \sin(x^3)$



$f(x) = \sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$

$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$ Look at powers of x
 even powers \Rightarrow even function $\sin(-x^2) = \sin(x^2)$

$\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$ odd powers \Rightarrow odd function
 $\sin(-x^3) = \sin(-x^3) = -\sin(x^3)$

3a) $4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots$ Geometric series where $r = \frac{1}{3}$ $a = 4$
 Sum = $\frac{a}{1-r} = \frac{4}{1-\frac{1}{3}} = \frac{4}{\frac{2}{3}} = \frac{4}{1} \cdot \frac{3}{2} = 6$

b) $1 - 1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$
 $= e^x = \sum_{n=0}^{\infty} \frac{(x)^n}{n!}$ where $x = -1$ so sum = $e^{-1} = \frac{1}{e}$

c) $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \dots$ Sum = $\cos \pi = -1$
 $= \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$ where $x = \pi$

4. For which x does $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n+1}}$ converge?

Ratio test $\lim_{n \rightarrow \infty} \left| \frac{(2x+3)^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(2x+3)^n} \right| = \lim_{n \rightarrow \infty} \frac{(2x+3)(\sqrt{n+1})}{\sqrt{n+2}}$ Goes to 1
 $= 2x+3$ For series to converge $|2x+3| < 1$

If $x = -2 \Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ which converges (include -2)

$-1 < 2x+3 < 1$

$-4 < 2x < -2$

$-2 < x < -1$

Test endpoints

If $x = -1 \Rightarrow \sum_{n=1}^{\infty} \frac{(1)^n}{\sqrt{n+1}}$ which diverges (p-series) (don't include -1)

$x = [-2, -1)$

Radius of convergence = $\frac{1}{2}$

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$$5. f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \dots \quad (\text{even function})$$

$$\lim_{x \rightarrow \infty} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2}}{x^6} = \lim_{x \rightarrow \infty} \frac{(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!}) - 1 - x^2 - \frac{x^4}{2}}{x^6}$$

$$= \lim_{x \rightarrow \infty} \frac{x^6 \left(\frac{1}{3!} + \frac{x^2}{4!} + \frac{x^4}{5!} + \dots \right)}{x^6} = \lim_{x \rightarrow \infty} \frac{1}{3!} + \frac{x^2}{4!} + \frac{x^4}{5!} = \frac{1}{6}$$

$$f(x) = e^{x^2} \quad f'(0) = 2 \quad f''(0) = 0 \quad f'''(0) = \frac{4!}{2!}$$

$$6. \frac{d}{dx} \int_{2x}^{3x} \frac{1}{\sqrt{1+t^2}} dt = \frac{d}{dx} (F(3x) - F(2x)) \quad \text{where } F'(t) = \frac{1}{\sqrt{1+t^2}}$$

Fundamental theorem of calculus

$$= 3F'(3x) - 2F'(2x) \quad (\text{chain rule})$$

$$= 3 \left(\frac{1}{\sqrt{1+(3x)^2}} \right) - 2 \left(\frac{1}{\sqrt{1+(2x)^2}} \right)$$

$$7a) \int_{-2}^2 (x+1)(x^2+1) dx = \int_{-2}^2 x^3 + x^2 + x + 1 dx$$

$$= \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-2}^2 = \left[\frac{2^4}{4} + \frac{2^3}{3} + \frac{2^2}{2} + 2 \right] - \left[\frac{(-2)^4}{4} - \frac{(-2)^3}{3} + \frac{(-2)^2}{2} - 2 \right]$$

$$= \frac{16}{3} + 4$$

$$b) \int \cos^2(3x) dx \quad u = 3x$$

$$= \frac{1}{3} \int \cos^2(u) dx \quad du = 3 dx$$

$$= \frac{1}{3} \int \frac{1 + \cos(2u)}{2} du = \frac{1}{3} \left[\frac{u}{2} + \frac{\sin(2u)}{4} \right] + c = \frac{1}{3} \left[\frac{3x}{2} + \frac{\sin(6x)}{4} \right] + c$$

$$\text{OR} = \int \frac{1 + \cos(6x)}{2} dx = \frac{x}{2} + \frac{\sin(6x)}{12} + c$$

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$$7c) \int \frac{dx}{1+x} \quad u=1+x \quad du=dx \quad \int \frac{du}{u} = \ln|u| + c = \ln|1+x| + c$$

$$c) \int \frac{dx}{1+x^2} = \arctan x + c$$

$$\text{Ex) } \int \frac{2x+1}{x^2+1} dx = \int \frac{2x}{x^2+1} dx + \int \frac{1}{x^2+1} dx$$

$$d) \int \frac{\ln(2x+4)}{x+2} dx \quad u = \ln(2x+4) \\ du = \frac{1}{2x+4} (2) dx = \frac{1}{x+2} dx$$

$$\int u du = \frac{u^2}{2} + c = \frac{(\ln(2x+4))^2}{2} + c$$

$$\text{OR} = \int \frac{\ln|2(x+2)|}{x+2} dx = \int \frac{\ln|2| + \ln|x+2|}{x+2} dx$$

TEST RATIO

with μ_0 known

$$\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

$$\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

$$0 \text{ is the value of } \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

$$1 > \frac{1}{\sigma^2} = \frac{1 + \dots}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

Conditionally constant

$$\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

$$\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

$$\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma^2} \quad \leftarrow \text{with } \mu_0 \text{ known}$$

Conditionally constant

TEST REVIEW

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1. f) $\sum_{n=1}^{\infty} \frac{\sin(n)}{\sqrt{n^2+1}} \leq \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}} \sim \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \rightarrow$ P series greater than 1.

Series is absolutely convergent

2. $f(x) = \sin(x^2)$ and $f(x) = \sin(x^3)$

even function
 $f(-x) = f(x)$

odd function
 $f(-x) = -f(x)$

$\sin(x^2) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$ even function

$\sin(x^3) = x^3 - \frac{x^9}{3!} + \frac{x^{15}}{5!} - \frac{x^{21}}{7!} + \dots$ odd function

3. a) $4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} \dots$ $\text{sum} = \frac{a}{1-r} \rightarrow \frac{4}{1-\frac{1}{3}} = \frac{4}{\frac{2}{3}} = \frac{12}{2} = \boxed{6}$

3. b) $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} \dots$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} \rightarrow \boxed{\frac{1}{e}}$

3. c) $1 - \frac{\pi^2}{2} + \frac{\pi^4}{24} - \frac{\pi^6}{720} + \dots$ $\cos(\pi) = \boxed{-1}$

4. For which x does $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n+1}}$ converge? $\rightarrow \lim_{n \rightarrow \infty} \left| \frac{(2x+3)^{n+1}}{\sqrt{n+2}} \cdot \frac{\sqrt{n+1}}{(2x+3)^n} \right| \rightarrow$

$\lim_{n \rightarrow \infty} \left| (2x+3) \cdot \frac{\sqrt{n+1}}{\sqrt{n+2}} \right| \xrightarrow{\text{goes to } 1} |2x+3| \stackrel{?}{\leq} 1 \rightarrow -1 \stackrel{?}{\leq} 2x+3 \stackrel{?}{\leq} 1 \rightarrow$

$-2 \stackrel{?}{\leq} x \stackrel{?}{\leq} -1 \rightarrow \sum_{n=1}^{\infty} \frac{(-4+3)^n}{\sqrt{n+1}} \rightarrow \text{converge!}$

$\sum_{n=1}^{\infty} \frac{(-2+3)^n}{\sqrt{n+1}} \rightarrow \text{diverges!}$

$-2 \leq x < -1$
 $x \in [-2, -1)$

TEST 2018

ausgabe 28

mit $\sum_{k=0}^{\infty} \frac{1}{2^k} = 2$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+1}} = 1$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+2}} = \frac{1}{2}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+3}} = \frac{1}{4}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+4}} = \frac{1}{8}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+5}} = \frac{1}{16}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+6}} = \frac{1}{32}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+7}} = \frac{1}{64}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+8}} = \frac{1}{128}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+9}} = \frac{1}{256}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+10}} = \frac{1}{512}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+11}} = \frac{1}{1024}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+12}} = \frac{1}{2048}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+13}} = \frac{1}{4096}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+14}} = \frac{1}{8192}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+15}} = \frac{1}{16384}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+16}} = \frac{1}{32768}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+17}} = \frac{1}{65536}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+18}} = \frac{1}{131072}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+19}} = \frac{1}{262144}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+20}} = \frac{1}{524288}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+21}} = \frac{1}{1048576}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+22}} = \frac{1}{2097152}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+23}} = \frac{1}{4194304}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+24}} = \frac{1}{8388608}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+25}} = \frac{1}{16777216}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+26}} = \frac{1}{33554432}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+27}} = \frac{1}{67108864}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+28}} = \frac{1}{134217728}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+29}} = \frac{1}{268435456}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+30}} = \frac{1}{536870912}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+31}} = \frac{1}{1073741824}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+32}} = \frac{1}{2147483648}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+33}} = \frac{1}{4294967296}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+34}} = \frac{1}{8589934592}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+35}} = \frac{1}{17179869184}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+36}} = \frac{1}{34359738368}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+37}} = \frac{1}{68719476736}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+38}} = \frac{1}{137438953472}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+39}} = \frac{1}{274877906944}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+40}} = \frac{1}{549755813888}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+41}} = \frac{1}{1099511627776}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+42}} = \frac{1}{2199023255552}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+43}} = \frac{1}{4398046511104}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+44}} = \frac{1}{8796093022208}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+45}} = \frac{1}{17592186044416}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+46}} = \frac{1}{35184372088832}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+47}} = \frac{1}{70368744177664}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+48}} = \frac{1}{140737488355328}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+49}} = \frac{1}{281474976710656}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+50}} = \frac{1}{562949953421312}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+51}} = \frac{1}{1125899906842624}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+52}} = \frac{1}{2251799813685248}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+53}} = \frac{1}{4503599627370496}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+54}} = \frac{1}{9007199254740992}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+55}} = \frac{1}{18014398509481984}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+56}} = \frac{1}{36028797018963968}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+57}} = \frac{1}{72057594037927936}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+58}} = \frac{1}{144115188075855872}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+59}} = \frac{1}{288230376151711744}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+60}} = \frac{1}{576460752303423488}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+61}} = \frac{1}{1152921504606846976}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+62}} = \frac{1}{2305843009213693952}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+63}} = \frac{1}{4611686018427387904}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+64}} = \frac{1}{9223372036854775808}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+65}} = \frac{1}{18446744073709551616}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+66}} = \frac{1}{36893488147419103232}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+67}} = \frac{1}{73786976294838206464}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+68}} = \frac{1}{147573952589676412928}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+69}} = \frac{1}{295147905179352825856}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+70}} = \frac{1}{590295810358705651712}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+71}} = \frac{1}{1180591620717411303424}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+72}} = \frac{1}{2361183241434822606848}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+73}} = \frac{1}{4722366482869645213696}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+74}} = \frac{1}{9444732965739290427392}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+75}} = \frac{1}{18889465931478580854784}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+76}} = \frac{1}{37778931862957161709568}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+77}} = \frac{1}{75557863725914323419136}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+78}} = \frac{1}{151115727451828646838272}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+79}} = \frac{1}{302231454903657293676544}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+80}} = \frac{1}{604462909807314587353088}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+81}} = \frac{1}{1208925819614629174706176}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+82}} = \frac{1}{2417851639229258349412352}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+83}} = \frac{1}{4835703278458516698824704}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+84}} = \frac{1}{9671406556917033397649408}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+85}} = \frac{1}{19342813113834066795298816}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+86}} = \frac{1}{38685626227668133590597632}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+87}} = \frac{1}{77371252455336267181195264}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+88}} = \frac{1}{154742504910672534362390528}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+89}} = \frac{1}{309485009821345068724781056}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+90}} = \frac{1}{618970019642690137449562112}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+91}} = \frac{1}{1237940039285380274899124224}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+92}} = \frac{1}{2475880078570760549798248448}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+93}} = \frac{1}{4951760157141521099596496896}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+94}} = \frac{1}{9903520314283042199192993792}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+95}} = \frac{1}{19807040628566084398385987584}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+96}} = \frac{1}{39614081257132168796771975168}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+97}} = \frac{1}{79228162514264337593543950336}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+98}} = \frac{1}{158456325028528675187087900672}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+99}} = \frac{1}{316912650057057350374175801344}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+100}} = \frac{1}{633825300114114700748351602688}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+101}} = \frac{1}{1267650600228229401496703205376}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+102}} = \frac{1}{2535301200456458802993406410752}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+103}} = \frac{1}{5070602400912917605986812821504}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+104}} = \frac{1}{10141204801825835211973625643008}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+105}} = \frac{1}{20282409603651670423947251286016}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+106}} = \frac{1}{40564819207303340847894502572032}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+107}} = \frac{1}{81129638414606681695789005144064}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+108}} = \frac{1}{162259276829213363391578010288128}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+109}} = \frac{1}{324518553658426726783156020576256}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+110}} = \frac{1}{649037107316853453566312041152512}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+111}} = \frac{1}{1298074214633706907132624082305024}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+112}} = \frac{1}{2596148429267413814265248164610048}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+113}} = \frac{1}{5192296858534827628530496329220096}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+114}} = \frac{1}{10384593717069655257060992658440192}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+115}} = \frac{1}{20769187434139310514121985316880384}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+116}} = \frac{1}{41538374868278621028243970633760768}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+117}} = \frac{1}{83076749736557242056487941267521536}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+118}} = \frac{1}{166153499473114484112975882535043072}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+119}} = \frac{1}{332306998946228968225951765070086144}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+120}} = \frac{1}{664613997892457936451903530140172288}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+121}} = \frac{1}{1329227995784915872903807060280344576}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+122}} = \frac{1}{2658455991569831745807614120560689152}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+123}} = \frac{1}{5316911983139663491615228241121378304}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+124}} = \frac{1}{10633823966279326983230456482242756608}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+125}} = \frac{1}{21267647932558653966460912964485513216}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+126}} = \frac{1}{42535295865117307932921825928971026432}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+127}} = \frac{1}{85070591730234615865843651857942052864}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+128}} = \frac{1}{170141183460469231731687303715884105728}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+129}} = \frac{1}{340282366920938463463374607431768211456}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+130}} = \frac{1}{680564733841876926926749214863536422912}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+131}} = \frac{1}{1361129467683753853853498429727072845824}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+132}} = \frac{1}{2722258935367507707706996859454145691648}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+133}} = \frac{1}{5444517870735015415413993718908291383296}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+134}} = \frac{1}{10889035741470030830827987437816582766592}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+135}} = \frac{1}{21778071482940061661655974875633165533184}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+136}} = \frac{1}{43556142965880123323311949751266331066368}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+137}} = \frac{1}{87112285931760246646623899502532662132736}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+138}} = \frac{1}{174224571863520493293247799005065324265472}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+139}} = \frac{1}{348449143727040986586495598010130648530944}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+140}} = \frac{1}{696898287454081973172991196020261297061888}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+141}} = \frac{1}{1393796574908163946345982392040522594123776}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+142}} = \frac{1}{2787593149816327892691964784081045188247552}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+143}} = \frac{1}{5575186299632655785383929568162090376495104}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+144}} = \frac{1}{11150372599265311570767859136324180752990208}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+145}} = \frac{1}{22300745198530623141535718272648361505980416}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+146}} = \frac{1}{44601490397061246283071436545296723011960832}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+147}} = \frac{1}{89202980794122492566142873090593446023921664}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+148}} = \frac{1}{178405961588244985132285746181186892047843328}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+149}} = \frac{1}{356811923176489970264571492362373784095686656}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+150}} = \frac{1}{713623846352979940529142984724747568191373312}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+151}} = \frac{1}{1427247692705959881058285969449495136382746624}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+152}} = \frac{1}{2854495385411919762116571938898990272765493248}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+153}} = \frac{1}{5708990770823839524233143877797980545530986496}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+154}} = \frac{1}{11417981541647679048466287755595961091061972992}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+155}} = \frac{1}{22835963083295358096932575511191922182123945984}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+156}} = \frac{1}{45671926166590716193865151022383844364247891968}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+157}} = \frac{1}{91343852333181432387730302044767688728495783936}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+158}} = \frac{1}{182687704666362864775460604089535377456991567872}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+159}} = \frac{1}{365375409332725729550921208179070754913983135744}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+160}} = \frac{1}{730750818665451459101842416358141509827966271488}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+161}} = \frac{1}{1461501637330902918203684832716283019655932542976}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+162}} = \frac{1}{2923003274661805836407369665432566039311865085952}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+163}} = \frac{1}{5846006549323611672814739330865132078623730171904}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+164}} = \frac{1}{11692013098647223345629478661730264157247460343808}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+165}} = \frac{1}{23384026197294446691258957323460528314494920687616}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+166}} = \frac{1}{46768052394588893382517914646921056628989841375232}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+167}} = \frac{1}{93536104789177786765035829293842113257979682750464}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+168}} = \frac{1}{187072209578355573530071658587684226515959365500928}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+169}} = \frac{1}{374144419156711147060143317175368453031918731001856}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+170}} = \frac{1}{748288838313422294120286634350736906063837462003712}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+171}} = \frac{1}{1496577676626844588240573268701473812127674924007424}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+172}} = \frac{1}{2993155353253689176481146537402947624255349848014848}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+173}} = \frac{1}{5986310706507378352962293074805895248510699696029696}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+174}} = \frac{1}{11972621413014756705924586149611790497021399392059392}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+175}} = \frac{1}{23945242826029513411849172299223580994042798784118784}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+176}} = \frac{1}{47890485652059026823698344598447161988085597568237568}$ \rightarrow $\sum_{k=0}^{\infty} \frac{1}{2^{k+177}} = \frac{1}{957809713$

TEST REVIEW

October 22, 2014

5. $f(x) = e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} + \dots$

$$\lim_{x \rightarrow 0} \frac{e^{x^2} - 1 - x^2 - \frac{x^4}{2}}{x^6} \rightarrow \lim_{x \rightarrow 0} \frac{(1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \frac{x^8}{4!} \dots) - 1 - x^2 - \frac{x^4}{2!}}{x^6} \rightarrow$$

$$\lim_{x \rightarrow 0} \frac{1}{3!} + \frac{x^2}{4!} + \frac{x^4}{5!} \dots = \boxed{\frac{1}{6}}$$

6. $\frac{d}{dx} \int_{2x}^{3x} \frac{1}{\sqrt{1+t^2}} dt \rightarrow$ FTC $\rightarrow \frac{d}{dx} (F(3x) - F(2x))$ where $F'(t) = \frac{1}{\sqrt{1+t^2}} \rightarrow$

$$\boxed{\frac{3}{\sqrt{1+(3x)^2}} - \frac{2}{\sqrt{1+(2x)^2}}}$$

7. a) $\int_{-2}^2 (x+1)(x^2+1) dx \rightarrow \int_{-2}^2 (x^3 + x^2 + x + 1) dx \rightarrow \left[\frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x \right]_{-2}^2 \rightarrow$

$$\left[\frac{2^4}{4} + \frac{2^3}{3} + \frac{2^2}{2} + 2 \right] - \left[\frac{(-2)^4}{4} + \frac{(-2)^3}{3} + \frac{(-2)^2}{2} - 2 \right] = \frac{16}{3} + 4 = \boxed{\frac{28}{3}}$$

7. b) $\int \cos^2(3x) dx \rightarrow \int \frac{1 + \cos 6x}{2} dx \rightarrow \boxed{\frac{x}{2} + \frac{\sin 6x}{12} + C}$

7. c) $\int \frac{dx}{1+x} \rightarrow \boxed{\ln |1+x| + C}$

7. d) $\int \frac{\ln(2x+4)}{x+2} dx \rightarrow u = \ln(2x+4)$
 $du = \frac{1}{2x+4} \cdot 2 dx \rightarrow du = \frac{1}{2x+2} dx \rightarrow$

$$\int u du \rightarrow \frac{u^2}{2} + C \rightarrow \boxed{\frac{[\ln(2x+4)]^2}{2} + C}$$

PROB. 28

$$\dots + \frac{0}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{x^2} + 1 = \sum_{n=1}^x \frac{1}{n^2} = (A)$$

$$\frac{1}{x} = \sum_{n=1}^x \frac{1}{n^2} - \sum_{n=1}^{x-1} \frac{1}{n^2} = \frac{1}{x^2} + \frac{1}{(x-1)^2} + \dots + \frac{1}{1^2} - \left(\frac{1}{(x-1)^2} + \dots + \frac{1}{1^2} \right)$$

$$\boxed{\frac{1}{x}} = \frac{1}{x^2} + \frac{1}{(x-1)^2} + \dots + \frac{1}{1^2}$$

$$\frac{1}{x+1} = (B) \quad \text{where } (A) - (B) = \frac{1}{x} - \frac{1}{x+1} = \frac{1}{x(x+1)}$$

$$\boxed{\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}}$$

$$\sum_{n=1}^x \left[\frac{1}{n} - \frac{1}{n+1} \right] = \sum_{n=1}^x \frac{1}{n} - \sum_{n=1}^x \frac{1}{n+1} = \sum_{n=1}^x \frac{1}{n} - \sum_{n=2}^{x+1} \frac{1}{n}$$

$$\boxed{\frac{1}{x(x+1)}} = \frac{1}{x} - \frac{1}{x+1} = \left[\frac{1}{x} - \frac{1}{x+1} \right] = \left[\frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \right]$$

$$\boxed{\frac{1}{x(x+1)} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)}} = \frac{x+1}{x(x+1)} - \frac{x}{x(x+1)} \quad (C)$$

$$\boxed{\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}} \quad (D)$$

$$\sum_{n=1}^x \frac{1}{n(n+1)} = \sum_{n=1}^x \left(\frac{1}{n} - \frac{1}{n+1} \right) = \left(\frac{1}{1} - \frac{1}{2} \right) + \left(\frac{1}{2} - \frac{1}{3} \right) + \dots + \left(\frac{1}{x} - \frac{1}{x+1} \right)$$

$$\boxed{\sum_{n=1}^x \frac{1}{n(n+1)} = 1 - \frac{1}{x+1} = \frac{x}{x+1}}$$