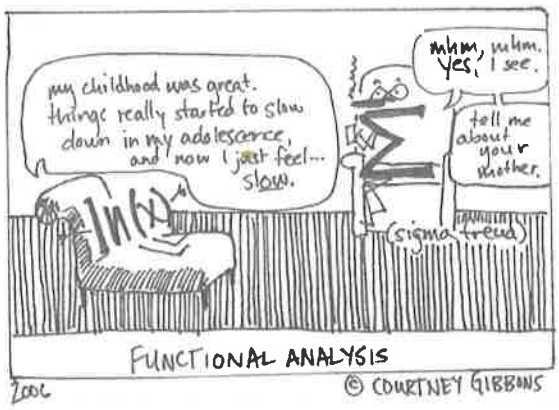
KEY

Math 126: Quiz 1
September 12, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Explain the difference between a *sequence* and a *series*

A sequence is an ordered list of real numbers

A series is a sum of terms in a sequence

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2. Give two examples of *geometric* series, one which converges and one which does not.

Conv Example: $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

Non Conv. $\frac{4}{3} + \frac{16}{9} + \frac{64}{27} + \cdots$

3. Determine whether the following sequences converge (and the limits, if they exist). Give a brief justification for your answers. (Write out the first few terms of each, if you find it helpful).

(a) $\left\{\frac{(-1)^{n+1}}{\sqrt{n}}\right\}_{n=1}^{\infty}$ $1, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{4}}, \ldots$

goes to 0. (denominator grows while numerator alternates

(b) $\left\{\frac{2n^2}{\sqrt{n^4+1}}\right\}_{n=1}^{\infty}$ polynomial → $\frac{\text{power of 2}}{\text{"power" of 2}}$

So compare leading coeff's

$L = \frac{2}{1} = 2$

(c) $\left\{\frac{n!}{(n-1)!}\right\}_{n=3}^{\infty}$ $\left\{\frac{3!}{2!}, \frac{4!}{3!}, \frac{5!}{4!}, \ldots\right\}$

$1 \quad = \{3, 4, 5, \ldots\}$

Algebraically this simplifies to n → ∞

(d) $\left\{\frac{\ln(n)}{n}\right\}_{n=1}^{\infty}$ logarithm ← grows slower than polynomial

↳ so the sequence tends to 0

4. (a) Write the following series in sigma notation.

$$\frac{1}{2}+\frac{1}{7}+\frac{1}{12}+\frac{1}{17}+\ldots$$

$\sum_{n=0}^{\infty} \frac{1}{5n+2}$ or $\sum_{n=1}^{\infty} \frac{1}{5n-3}$ or others.

(b) Does the above series converge? Why or why not?

No. The terms "plod" along, the denominator only ever growing by 5. Thus, it Diverges, like the Harmonic Series.

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(c) Write the following series in sigma notation, and determine whether or not it converges.

$$\frac{1}{2}-\frac{1}{7}+\frac{1}{12}-\frac{1}{17}+\ldots$$

$\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+2}$ This Converges as the terms are getting smaller & smaller, Closer to 0.

5. Determine the convergence (and sum) or divergence of each of the following series. Provide a justification for your answers. (Again, writing out the first few terms may help).

(a) $\sum_{n=1}^{\infty} \frac{3^n + 4^n}{5^n}$ $= \sum_{n=1}^{\infty} \frac{3^n}{5^n} + \frac{4^n}{5^n}$ Both are Geometric, with ratios < 1

$= \frac{3/5}{1-3/5} + \frac{4/5}{1-4/5} = \frac{3}{2} + 4 = 5.5$

(b) $\sum_{n=1}^{\infty} \frac{1}{n+3}$ $\hookrightarrow$ Diverges, as it is Harmonic

(c) $\sum_{n=1}^{\infty} \frac{2n^2}{\sqrt{n^4+1}}$ Since $\lim_{n\to\infty} \frac{2n^2}{\sqrt{n^4+1}} = 2 \neq 0$, this Series Diverges

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(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}$ $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \cdots = \frac{1}{1-(-1/4)} = \frac{4}{5}$

6. (Bonus) Alice and Bob are going to take turns tossing a fair coin, Alice goes first. Whoever gets the first head get \$1 from the other. Who has the advantage? What is the probability that that person will win?

Alice has the advantage, since she goes first

She wins with prob $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \cdots = \frac{1/2}{1-1/4}$

H, TTH, TTTTH ...

$= \frac{1/2}{3/4} = \frac{2}{3}$ ✓