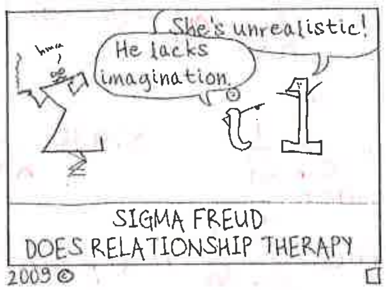
KEY

Math 126: Quiz 2
September 26, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Suppose that $\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} b_n$ are series such that $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, with c finite and $c \neq 0$. What conclusions can we draw about the two series.

In this case, the limit comparison test applies

$\sum a_n$ and $\sum b_n$ Both converge or Both Diverge.

2. Suppose that we have a series $\sum_{n=0}^{\infty} a_n$ for which $a_n \leq \frac{1}{n}$ for all n. What conclusions can we draw about $\sum_{n=0}^{\infty} a_n$?

We can make no conclusion. Since $\sum a_n \leq \sum \frac{1}{n}$, but $\sum \frac{1}{n}$ diverges, we get no information about $\sum a_n$

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3. Suppose that we have a series $\sum_{n=0}^{\infty} a_n$ for which $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L$. What conclusions can we draw based on the value of L?

$L < 1$: $\sum a_n$ is abs. Convergent

$L > 1$: $\sum a_n$ is Divergent

$L = 1$: We can make no conclusion about $\sum a_n$.

6 → Answers
9 - details

4. For each of the six series presented, tell whether the series converges absolutely, converges conditionally, or diverges. In *three* of the cases, carry out an appropriate test to justify your response.

Cond. Conv. (a) $\sum_{n=2}^{\infty} \frac{(-1)^n\,(2n+1)}{n\sqrt{n}}$

Alt Series with $\lim_{n\to\infty} a_n = 0 \Rightarrow$ Convergent

$\sum_{n=2}^{\infty} \frac{(2n+1)}{n\sqrt{n}} \sim \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ since $\lim_{n\to\infty} \left| \frac{\frac{2n+1}{n\sqrt{n}}}{\frac{1}{\sqrt{n}}} \right| = 2$

since $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ is a p-series with $p<1$, $\sum_{n=2}^{\infty} \frac{2n+1}{n\sqrt{n}}$ Diverges

Abs Conv. (b) $\sum_{n=0}^{\infty} \frac{\ln(n+1)}{2^n}$

Ratio test: $\lim_{n\to\infty} \left| \frac{\ln(n+2)}{2^{n+1}} \cdot \frac{2^n}{\ln(n+1)} \right| = \lim_{n\to\infty} \left| \frac{\ln(n+2)}{\ln(n+1)} \cdot \frac{1}{2} \right| = \frac{1}{2} < 1$

$\Rightarrow$ Series converges.

Abs Conv. (c) $\sum_{n=0}^{\infty} \frac{(-8)^n}{3^n n!}$

Ratio test: $\lim_{n\to\infty} \left| \frac{(-8)^{n+1}}{3^{n+1}(n+1)!} \cdot \frac{n!\,3^n}{(-8)^n} \right| = \lim_{n\to\infty} \left| \frac{-8}{3} \cdot \frac{1}{n+1} \right| = 0 < 1$

Abs Conv.

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(n+2)!}{2^{n^2}}$

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{(n+3)!}{2^{(n+1)^2}} \cdot \frac{2^{n^2}}{(n+2)!} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+3}{2^{2n+1}} \right| = 0 < 1$$

Abs conv.

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n+4}{4^n+3} \sim \sum \frac{3^n}{4^n}$ ← geom, $r = 3/4$

$$\lim_{n\to\infty} \left(\frac{\frac{3^n+4}{4^n+3}}{\frac{3^n}{4^n}} \right) = \lim_{n\to\infty} \frac{\frac{3^n+4}{3^n}}{\frac{4^n+3}{4^n}} = \frac{1+\frac{4}{3^n}}{1+\frac{3}{4^n}} = 1$$

⇒ both series behave the same
⇒ Both Converge

Div.

(f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\sqrt{2n^8+5}}{n^4+n^2} \right)^n$

Root test:

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left(\frac{\sqrt{2n^8+5}}{n^4+n^2} \right)^n}$$

$$= \lim_{n\to\infty} \frac{\sqrt{2n^8+5}}{n^4+n^2} = \sqrt{2} > 1 \Rightarrow \text{Divergence}$$

5. (From this weekend's Senior Written Exam in Mathematics) For which values of x does the series

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2}(x+2)^n$$

converge? Be sure to check endpoints!

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Ratio Test:

$$\lim_{n\to\infty} \left| \frac{(n+2)(x+2)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(n+1)(x+2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \cancel{\frac{n+2}{n+1}} \cancel{\frac{n^2}{(n+1)^2}} (x+2) \right| = |x+2| < 1$$

$$-1 < x+2 < 1$$

$$\Rightarrow -3 < x < 1$$

If $x=-3$, the series is $\sum_{n=1}^{\infty} \frac{n+1}{n^2}(-1)^n$ Alt, $a_n \to 0$, Conv.

$x=-1$ the series is $\sum_{n=1}^{\infty} \frac{n+1}{n^2} \sim \sum_{n=1}^{\infty} \frac{1}{n}$, Divergent.

$-3 \le x < 1$ or $x \in [-3, 1)$.