

KEY

Math 126: Quiz the Third

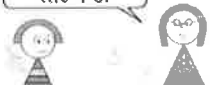
October 10, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!

Are you tired of adding C? spikedmath.com
© 2012

$$\int 4x^3 dx = x^4 + C$$

Don't forget the +C!



Then try these awesome alternatives!

Why use C when you can define P to be your constant.

$$\int 4x^3 dx = x^4 + P, \text{ where } P \text{ is an arbitrary constant.}$$

Subtract C instead.

$$\int 4x^3 dx = x^4 - C$$

Add C. Then add 42.

$$\int 4x^3 dx = x^4 + C + 42$$

Add any function of C whose range is all real numbers.

$$\int 4x^3 dx = x^4 + \tan(C), \text{ where } C \in (-\pi/2, \pi/2).$$

Add monkey.

$$\int 4x^3 dx = x^4 + \text{monkey}$$

Bonus points for drawing a monkey!

$$\int 4x^3 dx = x^4 + \text{🐵}$$

where 🐵 is an arbitrary constant.



That's not the answer in the teacher's guide!!

(or WebWork!)

1. (a) Find a series representation for

$$f(x) = \frac{4}{2+x} = \frac{a}{1-r}$$

List at least the first five non-zero terms.

$$f(x) = \frac{4}{2+x} = \frac{2}{1 + \left(\frac{x}{2}\right)} \quad \text{geom, ratio } -\frac{x}{2}, a=2$$

$$f(x) = 2 - x + \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{8} - \dots$$

(b) Find the *fourth* derivative of $f(x) = \frac{4}{2+x}$ evaluated at $x=0$.

$$f^{(4)}(0) = 4! \cdot c_4 = \frac{4!}{8} = 3$$

2. (a) Find a series representation for $f(x) = e^{(-2x^3)}$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$e^{(-2x^3)} = 1 - 2x^3 + \frac{(2x^3)^2}{2!} - \frac{(2x^3)^3}{3!} + \dots = \sum_{n=0}^{\infty} \frac{(-2x^3)^n}{n!}$$

(b) Use your series to determine

$$\lim_{x \rightarrow 0} \frac{e^{(-2x^3)} - 1 + 2x^3}{3x^6}$$

$$= \lim_{x \rightarrow 0} \frac{(1 - 2x^3 + \frac{(2x^3)^2}{2!} - \frac{(2x^3)^3}{3!}) - 1 + 2x^3}{3x^6}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{4x^6}{2!} - \frac{8x^9}{3!} + \frac{16x^{12}}{4!} - \dots}{3x^6} = \lim_{x \rightarrow 0} \frac{\frac{4}{2!} - \frac{8x^3}{3!} + \frac{16x^6}{4!} - \dots}{3} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

3. Find a series representation for $f(x) = \frac{\sin(x)}{x}$. Write your answer in Sigma notation.

$$f(x) = \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$f(x) = \frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

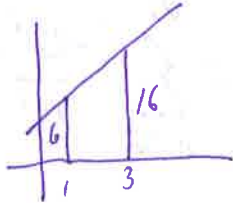
4. Use your series to find the antiderivative $\int \frac{\sin(x)}{x} dx$. Write your answer in Sigma notation.

$$\int \frac{\sin x}{x} dx = x - \frac{x^3}{3 \cdot 3!} + \frac{x^5}{5 \cdot 5!} - \frac{x^7}{7 \cdot 7!} + \dots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!} + C$$

5. Determine $\int_1^3 5x + 1 dx$ using

(a) Geometry

Note:
Not to Scale



$$\begin{aligned} \text{Area} &= \frac{1}{2} (b_1 + b_2) \cdot h \\ &= \frac{1}{2} (6 + 16) \cdot 2 = \underline{\underline{22}} \end{aligned}$$

(b) Calculus

$$\int_1^3 5x + 1 dx = \left. \frac{5}{2} x^2 + x \right|_1^3$$

$$= \frac{5 \cdot 9}{2} + 3 - \left(\frac{5}{2} + 1 \right) = \frac{51}{2} - \frac{7}{2} = \frac{44}{2} = \underline{\underline{22}}$$

6. Determine

$$\frac{d}{dx} \int_3^{x^2} \sqrt{t^3 + 1} dt$$

$$\frac{d}{dx} \int_3^{x^2} \sqrt{t^3 + 1} dt = \frac{d}{dx} (G(x^2) - G(3)) \quad \text{where } G'(t) = \sqrt{t^3 + 1}$$

$$= 2x G'(x^2) - 0$$

$$= 2x \sqrt{x^6 + 1}$$