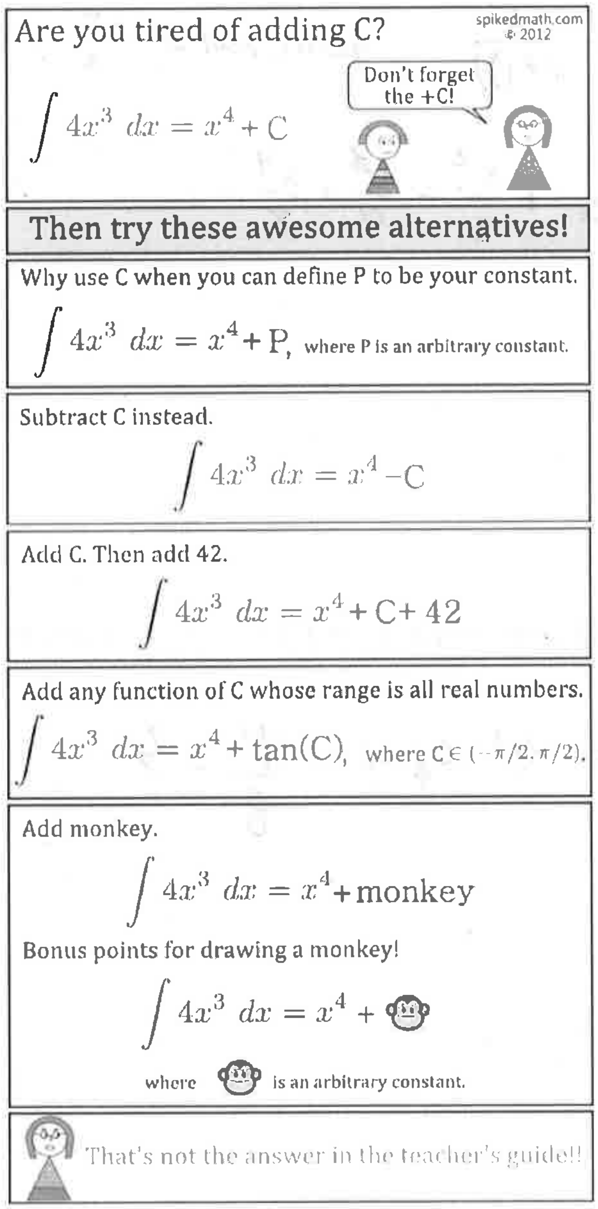
KEY

Math 126: Quiz the Third

October 10, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



(or WebWork!)

1. (a) Find a series representation for

$$f(x) = \frac{4}{2+x}. \quad = \frac{a}{1-r}$$

List at least the first five non-zero terms.

$f(x) = \frac{4}{2+x} = \frac{2}{1+\left(\frac{x}{2}\right)}$ geom, ratio $-\frac{x}{2}$, $a = 2$

$f(x) = 2 - x + \frac{x^2}{2} - \frac{x^3}{4} + \frac{x^4}{8} - \cdots$

(b) Find the *fourth* derivative of $f(x) = \dfrac{4}{2+x}$ evaluated at $x = 0$.

$f''''(0) = 4! \cdot c_4 = \frac{4!}{8} = 3$

2. (a) Find a series representation for $f(x) = e^{(-2x^3)}$.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^{(-2x^3)} = 1 - 2x^3 + \frac{(2x^3)^2}{2!} - \frac{(2x^3)^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{(-2x^3)^n}{n!}$$

(b) Use your series to determine

$$\lim_{x\to 0} \frac{e^{(-2x^3)} - 1 + 2x^3}{3x^6}$$

$$= \lim_{x\to 0} \frac{\left(1 - 2x^3 + \frac{(2x^3)^2}{2!} - \frac{(2x^3)^3}{3!}\right) - 1 + 2x^3}{3x^6}$$

$$= \lim_{x\to 0} \frac{\frac{4x^6}{2!} - \frac{8x^9}{3!} + \frac{16x^{12}}{4!} - \cdots}{3x^6} = \lim_{x\to 0} \frac{\frac{4}{2!} - \frac{8x^3}{3!} + \frac{16x^6}{4!} \cdots}{3} = \frac{4}{6} = \boxed{\frac{2}{3}}$$

3. Find a series representation for $f(x) = \dfrac{\sin(x)}{x}$. Write your answer in Sigma notation.

$$\cancel{f(x) =} \quad \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$f(x) = \frac{\sin x}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n+1)!}$$

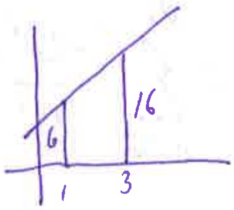
4. Use your series to find the antiderivative $\displaystyle\int \frac{\sin(x)}{x}\,dx$. Write your answer in Sigma notation.

$$\int \frac{\sin x}{x}\,dx = x - \frac{x^3}{3\cdot 3!} + \frac{x^5}{5\cdot 5!} - \frac{x^7}{7\cdot 7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)(2n+1)!} + C$$

5. Determine $\int_1^3 5x + 1\, dx$ using

(a) Geometry

Note: NOT to Scale



$$\text{Area} = \frac{1}{2}(b_1 + b_2)\cdot h$$
$$= \frac{1}{2}(6 + 16)\cdot 2 = \underline{\underline{22}}$$

(b) Calculus

$$\int_1^3 5x+1\, dx = \frac{5}{2}x^2 + x \Big|_1^3$$
$$= \frac{5\cdot 9}{2} + 3 - \left(\frac{5}{2}+1\right) = \frac{51}{2} - \frac{7}{2} = \frac{44}{2} = \underline{22}$$

6. Determine

$$\frac{d}{dx}\int_3^{x^2} \sqrt{t^3+1}\, dt$$

$$\frac{d}{dx}\int_3^{x^2}\sqrt{t^3+1}\, dt = \frac{d}{dx}\left(G(x^2) - G(3)\right) \quad \text{where } G'(t) = \sqrt{t^3+1}$$
$$= 2x\, G'(x^2) - 0$$
$$= 2x\sqrt{x^6+1}$$