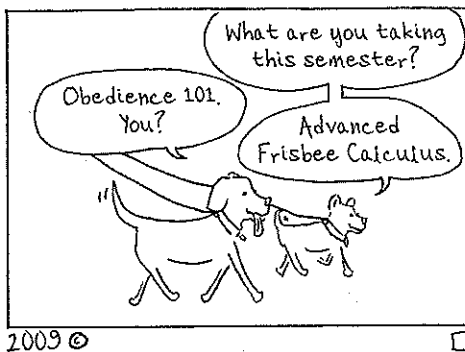


# KEY

## Math 126: Quiz 4

November 7, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!



1. Which method would you use to solve each of the following integrals? A one-sentence answer will suffice. 'Wolfram Alpha' is not a one-sentence answer.

(a)  $\int \frac{x dx}{\sqrt{4-9x^2}}$  Substitution  $u = 4 - 9x^2$

(b)  $\int \frac{dx}{1-9x^2}$  either Trig Sub  $x = \frac{1}{3} \sin \theta$   
or Partial fraction  $\int \frac{dx}{(-3x)(1+3x)}$

(c)  $\int \ln(x^2 + x) dx$  "Dry" IBP.

(d)  $\int \frac{dx}{\sqrt{4-9x^2}}$  Trig Sub  $x = \frac{2}{3} \sin \theta$

Find FOUR of the following FIVE antiderivatives. Please clearly indicate which one is not to be graded. If you fail to indicate, I will choose one at random.

2.

$$\int \frac{dx}{\sqrt{9-x^2}} \quad \begin{array}{l} \text{Let } x = 3 \sin \theta \\ dx = 3 \cos \theta d\theta \end{array}$$

$$= \int \frac{\cancel{dx} 3 \cos \theta d\theta}{\sqrt{9-9 \sin^2 \theta}} = \int \frac{3 \cos \theta d\theta}{3 \cos \theta} = \int 1 d\theta$$

$$= \theta + C$$

$$= \arcsin \frac{x}{3} + C$$

3.

$$\int \frac{5 dx}{x^2 - x - 6} = \int \frac{5}{(x-3)(x+2)} dx$$

$$\frac{5}{(x-3)(x+2)} = \frac{A}{x-3} + \frac{B}{x+2} = \frac{1}{x-3} - \frac{1}{x+2}$$

$$5 = A(x+2) + B(x-3)$$

$$x = -2 \Rightarrow 5 = B(-2-3) = -5B \Rightarrow B = -1$$

$$x = 3 \Rightarrow 5 = A(3+2) = 5A \Rightarrow A = 1$$

$$\text{So } \int \frac{5 dx}{x^2 - x - 6} = \int \frac{dx}{x-3} - \int \frac{dx}{x+2}$$

$$= \ln|x-3| - \ln|x+2| + C$$

4.

$$\int \cos^3(2x) \tan(2x) dx = \int \cos^3(2x) \frac{\sin(2x)}{\cos(2x)} dx$$

$$= \int \cos^2(2x) \sin(2x) dx$$

$$\text{let } u = \cos 2x$$

$$du = -2 \sin 2x dx$$

$$-\frac{1}{2} \int u^2 du = -\frac{u^3}{6} + C = \frac{-\cos^3(2x)}{6} + C$$

5.

$$\int x e^{3x} dx \rightarrow$$

$$uv - \int v du$$

$$\frac{1}{3} x e^{3x} - \int \frac{1}{3} e^{3x} dx$$

IBP:

$$u = x \quad dv = e^{3x} dx$$

$$du = dx$$

$$v = \frac{1}{3} e^{3x}$$

$$\frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x} + C$$

6.

$$\int \frac{x^3}{x+1} dx = \int x^2 - x + 1 - \frac{1}{x+1} dx$$

$$= \frac{x^3}{3} - \frac{x^2}{2} + x - \ln|x+1| + C$$

method

A

$$\begin{array}{r} x^2 - x + 1 - \frac{1}{x+1} \\ x+1 \overline{) x^3} \\ \underline{x^3 + x^2} \phantom{+ 1} \\ -x^2 \phantom{+ 1} \\ \underline{-x^2 - x} \phantom{+ 1} \\ x \phantom{+ 1} \\ \underline{-x + 1} \\ 1 \end{array}$$

$$\text{Method B} \quad \text{let } u = x+1$$

$$du = dx$$

$$\int \frac{(u-1)^3}{u} du = \int \frac{u^3 - 3u^2 + 3u - 1}{u} du$$

$$= \int u^2 - 3u + 3 - \frac{1}{u} du$$

$$= \frac{u^3}{3} - \frac{3}{2} u^2 + 3u - \ln|u| + C$$

$$= \frac{(x+1)^3}{3} - \frac{3}{2} (x+1)^2 + 3(x+1) - \ln|x+1| + C$$

7. Which of the following is  $\int \frac{x^3}{\sqrt{4x^2+9}} dx$  (Do NOT try to compute the integral!). Explain your choice.

- (a)  $\frac{1}{24}(2x-9)\sqrt{4x^2+9}$   
 (b)  $\frac{1}{24}(2x^2-9)\sqrt{4x^2+9}$   
 (c)  $\frac{1}{24}(2x^3-9)\sqrt{4x^2+9}$   
 (d)  $\frac{1}{24}\sqrt{4x^2+9}$

$$\int \frac{x^3}{\sqrt{4x^2+9}} dx$$

is about  $\int x^2 dx \sim \frac{x^3}{3} + C$

So we look  
for an effective  
power of 3  
on  $x$ .

8. (Bonus) Which topics of application (biology, chemistry, economics, mathematics, probability, or others) would you like to see covered in the remaining weeks of the semester?

9. (Bonus) Give a creative nomenclature for a 'group of integrals'.