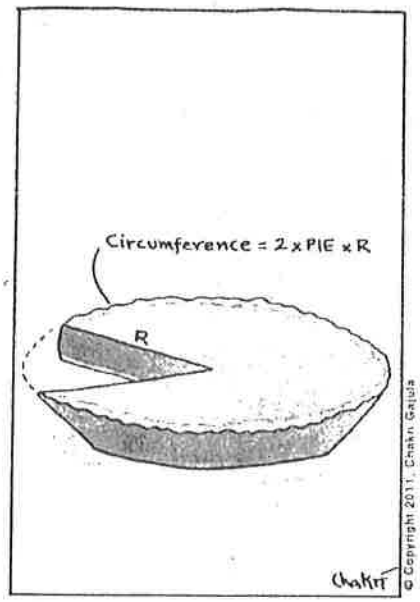
KEY

# Math 126: Quiz 5

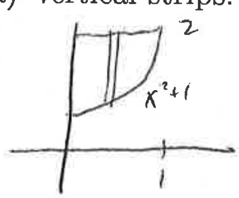
November 21, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic only (ie, no plotting). PLEASE READ ALL DIRECTIONS CAREFULLY!




1. Consider the area bound by the curve $y = x^2 + 1$, and the lines $x = 0$ and $y = 2$. Calculate this area using... $x = \sqrt{y-1}$

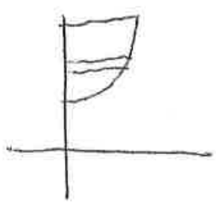
   (a) vertical strips.




$$A = \int_0^1 2-(x^2+1)\,dx = \int_0^1 1-x^2\,dx$$

$$= x - \frac{x^3}{3}\Big|_0^1 = 2/3$$

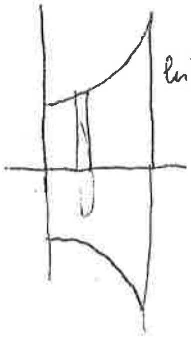
   (b) horizontal strips.



$$A = \int_1^2 \sqrt{y-1}\,dy = \frac{2}{3}(y-1)^{3/2}\Big|_1^2 = \frac{2}{3}\left((2-1)^{3/2} - (1-1)^{3/2}\right)$$

$$= \frac{2}{3}$$

2. Find the volume generated by rotating the area bound by $y = e^x$, the coordinate axes, and the line $x = \ln 3$ about the $x$-axis.




$$V = \int \pi y^2 \, dx = \int_0^{\ln 3} \pi (e^x)^2 \, dx$$

$$= \int_0^{\ln 3} \pi e^{2x} \, dx = \pi \left. \frac{e^{2x}}{2} \right]_0^{\ln 3}$$

$$= \pi \left( \frac{e^{2\ln 3}}{2} - \frac{e^0}{2} \right)$$

$$= \pi \left( \frac{9}{2} - \frac{1}{2} \right) = 4\pi$$

3. A tetrahedron has an equilateral triangle as its base, and parallel cross sections that are also equilateral triangles. If the base and the height are both 6, find the volume of the tetrahedron. (The formula for the area of an equilateral triangle is $\frac{\sqrt{3}}{4}s^2$). You can get partial credit here by applying a formula; for full credit, you should use calculus.


$$V_{slab} = \frac{\sqrt{3}}{4} s^2 w$$

$$\frac{s}{6-y} = \frac{6}{6} \Rightarrow s = 6 - y$$

$$V_{total} = \int_0^6 \frac{\sqrt{3}}{4} s^2 \, dy$$

$$\frac{\sqrt{3}}{4} \int_0^6 (6-y)^2 \, dy = \frac{\sqrt{3}}{4} \left. \frac{(6-y)^3}{-3} \right|_0^6$$

$$= \frac{\sqrt{3}}{4} \cdot \frac{6^3}{3}$$

$$= \frac{1}{3} \left( \frac{\sqrt{3}}{4} 6^2 \right) 6$$

$$\frac{1}{3} \quad B \quad h$$

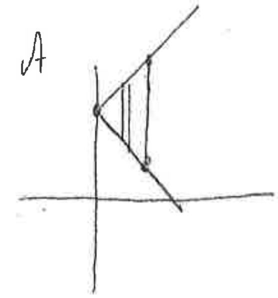
4. Consider the area bound by the lines $y = 5-x$, $y = 5+2x$ and $x = 3$. Below are four 'volume problems' and four integrals. Match the integral to the relevant problem. WARNING: There is one outlier in each group.

I. Rotate the area about the $y$-axis

II. Rotate the area about the $x$-axis

III. Rotate the area about the line $y = 2$

IV. Rotate the area about the line $x = 3$.




a) $\int_0^3 2\pi((5+2x)-(5-x))(3-x)\,dx$

b) $\int_0^3 2\pi(5+2x)-(5-x))x\,dx$

c) $\int_0^3 \pi((5+2x)-(5-x)))^2\,dx$

d) $\int_0^3 \pi((5+2x)^2-(5-x)^2)\,dx$

$dx \to$ vert strips

I rotate about $y$-axis $\to$ Shells

$V = \int 2\pi r h w = \int 2\pi x \left((5+2x)-(5-x)\right)dx$

I $\leftrightarrow$ (b)

II rotate about $x$-axis $\to$ Washers

~~V = ∫π(R~~ $V = \int \pi (R_{out}^2 - R_{in}^2)\,dx$

$= \int \pi \left((5+2x)^2 - (5-x)^2\right)dx$

II $\leftrightarrow$ (d)

III rotate about $y=2$ $\to$ Washers : $\int \pi\, (5+2x-2)^2 - (5-x-2)^2\,dx$

NO MATCH

IV rotate about $x=3$ $\to$ Shells : $\int 2\pi\left((5+2x)-(5-x)\right)(3-x)\,dx$



IV $\leftrightarrow$ (a)

5. (a) Find the length of the curve given by the function $f(x) = 4x^{3/2}$ from $(1, 4)$ to $(9, 108)$

$f'(x) = \frac{4 \cdot 3}{2} x^{1/2} = 6x^{1/2}$

$$\text{Length} = \int_1^9 \sqrt{1 + (6\sqrt{x})^2}\, dx$$

$$= \int_1^9 \sqrt{1+36x}\, dx = \frac{2}{3} \frac{(1+36x)^{3/2}}{36} \Big|_1^9$$

$$= \frac{2}{3}\left(\frac{(325)^{3/2} - (37)^{3/2}}{36}\right)$$

(b) Set up, but don't evaluate, the integral for the surface area generated when this curve is rotated about the $x$-axis.

$$S.A. = \int 2\pi r\, ds = \int_1^9 2\pi \left(4x^{3/2}\right) \sqrt{1 + 36x}\, dx$$

(c) Set up, but don't evaluate, the integral for the surface area generated when this curve is rotated about the $y$-axis.

$$SA = \int 2\pi r\, ds = \int_1^9 2\pi\, x \sqrt{1+36x}\, dx$$

$$= \int_4^{108} 2\pi \left(\frac{y}{4}\right)^{2/3} \sqrt{1 + \left(\frac{2}{3}\left(\frac{y}{4}\right)^{-1/3} \cdot \frac{1}{4}\right)^2}\, dy$$