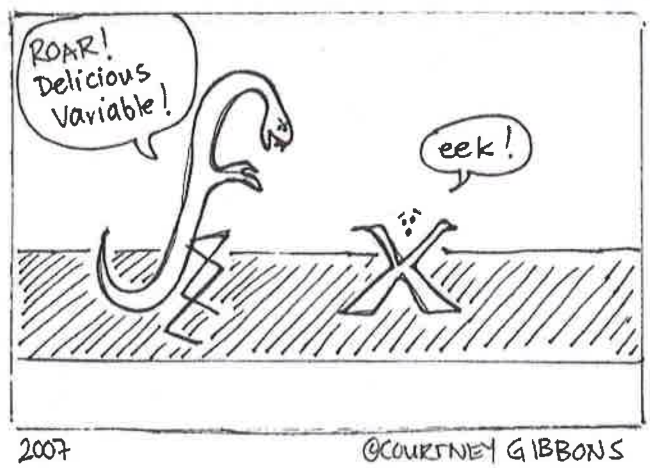
KEY

Math 126: Quiz 6
December 10, 2014

You have the remainder of the hour to complete this closed-book, closed-notes, closed-colleague quiz. You may use a calculator for arithmetic and trig/exponential functions only, i.e., no plotting and no calculus. PLEASE READ ALL DIRECTIONS CAREFULLY and JUSTIFY YOUR ANSWERS!



1. Of the three functions listed below, one is a probability density function, one is a cumulative distribution function, and one is neither. Which is which? Explain your reasoning.

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   (a) $f(x) = 2e^{-2x}$ $(x \geq 0)$ pdf $\int_0^{\infty} 2e^{-2x}\,dx =$
   (b) $f(x) = e^{-2x}$ $(x \geq 0)$ neither $\to$ $\int_0^{\infty} e^{-2x}\,dx \neq 1$
   (c) $f(x) = 1 - e^{-2x}$ $(x \geq 0)$ cdf $\lim_{x\to\infty} f(x) = 1$

2. (Note: This question has SEVEN parts) A variable x is modeled by the probability density function $f(x) = \dfrac{c}{x^2}$ for $x \geq 3$.

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(a) Find the value of c that makes $f(x)$ a p.d.f.

$$\int_3^{\infty} \frac{c}{x^2}\,dx = 1$$

$$\lim_{b \to \infty} -\frac{c}{\cancel{x}}\Big|_3^{b} = \lim_{b \to \infty} \frac{-c}{b} + \frac{c}{3} = 1$$

$$c = 3$$

(b) Find the probability that $4 \leq x \leq 6$.

$$\int_4^6 \frac{3}{x^2}\,dx = \frac{-3}{x}\Big|_4^6 = -\frac{1}{2} + \frac{3}{4} = \frac{1}{4}$$

(c) Find the probability that $x \leq 10$.

$$\int_3^{10} \frac{3}{x^2}\,dx = \frac{-3}{x}\Big|_3^{10} = \frac{-3}{10} + 1 = \frac{7}{10}$$

(d) Find the cumulative distribution function associated with f.

$$F(a) = \int_{-\infty}^{a} f(x)\,dx = \int_{3}^{a} \frac{3}{x^2}\,dx = \left.\frac{-3}{x}\right|_{3}^{a} = 1 - \frac{3}{a}$$

(e) Find the *median* value of x.

$$0.5 = \int_{3}^{a} \frac{3}{x^2}\,dx = 1 - \frac{3}{a}$$

$$\frac{3}{a} = 0.5 \Rightarrow a = 6$$

(f) Does x have a finite *mean*? Explain.

Mean: $\displaystyle\int_{-\infty}^{\infty} x f(x)\,dx = \int_{3}^{\infty} \frac{3x}{x^2}\,dx = \int_{3}^{\infty} \frac{3}{x}\,dx$

$$= 3\ln x \Big|_{3}^{\infty} \rightarrow \infty$$

no, it does not!

(g) Write a scenario for a variable x that might be modeled by $f(x)$. Be sure to account for the fact that $x \geq 3$. Creativity welcome!

We are waiting at a doctors office for the next patient to be called. Each patient will take at least 3 minutes to be seen. Our variable x measures the time between patients.

3. Find y as a function of x if $\dfrac{dy}{dx} = 2xy$ and $y(1) = 1$.

$$\frac{dy}{dx} = 2xy \rightarrow \frac{dy}{y} = 2x\,dx \rightarrow \ln|y| = x^2 + C$$

$$y = Be^{x^2}$$

$$y(1) = 1 \Rightarrow$$

$$1 = Be \Rightarrow B = 1/e$$

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So $y = \frac{1}{e} e^{x^2}$ or $y = e^{x^2 - 1}$

4. Hungary is one of the few countries in the world with a world with a decreasing population. Every year, 2 percent of the current population either dies off or leaves, while only 150,000 new people are born or move there. Hungary's current population is 10,000,000. (1) Set up a differential equation to determine the population as a function of time. (2) When will the population dip below 9,000,000? (3) What is the projected long term population?

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(1) $$\frac{dP}{dt} = 150{,}000 - .02P = -.02(P - 7{,}500{,}000)$$

$$\frac{dP}{P - 7{,}500{,}000} = -.02\,dt$$

$$\ln|P - 7{,}500{,}000| = \cancel{}.02t + C$$

$$P = 7{,}500{,}000 + Be^{.02t}$$

$$10{,}000{,}000 = 7{,}500{,}000 + Be^{0} \rightarrow B = 2{,}500{,}000$$

$$P = 7{,}500{,}000 + 2{,}500{,}000\, e^{-.02t}$$

solve for t

(2) $\rightarrow$ $9\text{ mil} = 7.5\text{ mil} + 2.5\text{ mil}\; e^{-.02t}$

$$t = \frac{\ln\left(\frac{1.5\text{ mil}}{2.5\text{ mil}}\right)}{-.02} = \underline{25.5\text{ years}}$$

(3) Long term population $\rightarrow$ 7,500,000

5. (Bonus) Write a caption for the cartoon on the front.